AN AXIOMATIC DESIGN APPROACH
TO PASSENGER ITINERARY ENUMERATION IN RECONFIGURABLE TRANSPORTATION SYSTEMS

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ABSTRACT

Transportation systems represent a critical infrastructure upon which nations' economies and national security depend. As infrastructure systems, they must be planned and operated to accommodate the uncertain and continually evolving needs of their passengers and freight. These changes represent not just changes in state - or system behavior - but also changes in system architecture. New routes and destinations are continually added and new modes of transport are introduced to realize them. Such changes occur in the planning time scale when the transportation is intentionally expanded, but also in the operational time scale when, for example, buses and trains breakdown. As such, transportation systems meet the Axiomatic Design classification of large flexible systems where many functional requirements not only evolve over time, but also can be fulfilled by one or more design parameters. This paper builds upon a recent work in which Axiomatic Design was used to develop a theory of degrees of freedom in manufacturing systems for their reconfigurable design and operation. The theory is specialized here to reconfigurable transportation systems. The methodological developments are then demonstrated on a small subsection of the Mexico City transportation system to demonstrate its wide ranging utility in reconfigurability decision-making at the planning and operations time scales.

Keywords: Axiomatic Design, transportation paths, transportation itineraries, Mexico City transportation system, re-configurability, resilience, reconfigurable transportation systems, resilient transportation systems.

1 INTRODUCTION

Transportation systems represent a critical infrastructure upon which nations' economies and national security depend. In the 1990s, transportation systems the world over became increasingly strained by the continually evolving needs of a growing population that has trended towards concentrating in cities for the past 100 years [de Weck *et al.*, 2011]. One particularly pertinent problem is the need to quickly find ways to reallocate and adjust the capacity and capabilities of transportation resources to the variants that need them most. Another key challenge is the transportation system's resilience in the face of unplanned disturbances, events, or disasters. Re-configurability and resilience drivers can be found to varying degrees in many of the modes of transport: air, ship, rail, and road. Recently, decentralized reconfiguration strategies for reconfigurable transportation systems have emerged [Vallee *et al.*, 2011]. In order to achieve and support these solutions, it becomes necessary to model the evolution of the system architecture. The realization of these incremental changes requires decisions to be made in the operations and planning of transportation systems. This requirement causes a multi-dimensional engineering management problem which stakeholders have to find ways to address. To fulfill these needs, reconfigurable transportation systems are proposed as a possible solution. They are defined as:

Definition 1. Reconfigurable Transportation System: A system designed at the outset for rapid change in structure, in order to quickly adjust capacity and functionality in response to sudden changes in stakeholder requirements. Reconfigurable transportation systems are those in which new capabilities are added only when needed, and the system is not over-designed with capabilities that may be left unused.

This paper uses an Axiomatic Design approach called transportation degrees of freedom to enumerate the number of passenger itineraries in reconfigurable transportation systems; transportation systems with variable system architecture. The enumeration of passenger itineraries, and more generically paths through a network, has long been associated with network reliability and resilience [de Silva *et al.*, 2011; Rai and Kumar, 1986; Khan and Singh, 1980]. Here, the Axiomatic Design based approach serves two additional purposes. First, the enumerated passenger itineraries are set in terms of the evolving system architecture variables in both function and form. Second, it bridges the traditionally graph theoretic approaches to the engineering design community.

The remainder of the paper proceeds as follows. Section II provides the background to the methodological developments with brief introductions to graph theory [van Steen, 2010; Lewis, 2009; Newman 2010], Axiomatic Design for large flexible systems [Suh, 2001], and production degrees of freedom [Farid, 2007; 2008; Farid and McFarlane, 2006a]. Section III then reframes previous work on production degrees of freedom [Farid, 2007, 2008; Farid and McFarlane, 2006a] into a transportation system context. Next, Section IV enumerates passenger itineraries as a measure called passenger degrees of freedom upon this foundation. Section V illustrates the methodological developments on a small subsection of the

Mexico City transportation system. Section VI describes the re-configurability applications of these measures in the planning and operations time scales. Section VII concludes the work and proposes avenues for future work.

2 BACKGROUND

This section summarizes the methodological developments found in the existing literature in order to provide a foundation for the enumeration of passenger itineraries in the next section. The discussion proceeds in three steps. Section 2.1 gives a brief introduction to graph theory while Section 2.2. introduces the application of Axiomatic Design for large flexible systems to transportation systems. Section 2.3 then discusses a taxonomy of transportation system degrees of freedom as presented in earlier work.

2.1 GRAPH THEORY IN TRANSPORTATION NETWORKS

Graph theory is a long established field of mathematics with applications in many fields of science and engineering where artifacts are transported between physical locations [van Steen, 2010; Lewis, 2009; Newman 2010]. A number of definitions from this field are introduced later in the discussion.

Definition 2. [van Steen, 2010] A graph: $G= \{V, E\}$, consists of a collection of nodes V and a collection of edges E. Each edge $e \in E$ is said to join two nodes, which are called its end points. If e joins $v_1, v_2 \in V$, we write $e=\langle v_1, v_2 \rangle$. Nodes v_1 and v_2 in this case are said to be adjacent. Edge e is said to be incident with nodes v_1 and v_2, respectively.

Definition 3. [van Steen, 2010] A directed graph (digraph): D, consists of a collection of nodes V, and a collection of arcs A, for which $D = \{V, A\}$. Each arc $a=\langle v_1, v_2 \rangle$ is said to join node $v_1 \in V$ to another (not necessarily distinct) node v_2. Vertex v_1 is called the tail of a, whereas v_2 is its head.

Definition 4. Adjacency matrix: A, is binary and of size $\sigma(V)$ x $\sigma(V)$ and its elements are given by:

$$A(i,j) = \begin{cases} 1 & if \langle v_i, v_j \rangle exists \\ 0 & otherwise \end{cases} \quad [1]$$

where the operator $\sigma()$ gives the size of a set. Interestingly, $A^N(i,j)$ represents the number of traveler itineraries of n-steps between origin i and destination j [Newman 2010].

Definition 5. [van Steen, 2010] Incidence matrix: M of size $\sigma(V)$ x $\sigma(A)$ is given by:

$$M(i,j) = \begin{cases} -1 & if \ vertex \ v_i \ is \ the \ head \ of \ arc \ a_j \\ 1 & if \ vertex \ v_i \ is \ the \ tail \ of \ arc \ a_j \\ 0 & otherwise \end{cases} \quad [2]$$

While graph theory for decades has presented a useful abstraction of transportation networks for operations research, it has limitations from an engineering design and systems engineering perspective. "Interestingly, the fraction of bona fide engineers pursuing this approach has remained relatively small; it is mostly mathematicians, physicists and biologists who pursue this particular view of complex systems. This may be because of the emphasis on analyzing systems 'as they are' rather than on building systems that do not yet exist. It may also be that engineers have to focus on technical details and many of them remain suspicious of highly abstracted mathematical representations of systems such as system graph representations, where all nodes are essentially treated as equal" [de Weck, 2011]. The above definitions focus on the abstract form of the transportation network and less so the transportation functions itself. Furthermore, how the function is realized is not explicitly stated. Unless generalized, such graph theoretic approaches are likely to have limitations in systems of heterogeneous function and form. Furthermore, because the system function and its realizing form has been abstracted away, such approaches may not straightforwardly lend themselves to active control solutions that implement reconfigurable transportation system architectures.

2.2 AXIOMATIC DESIGN FOR LARGE FLEXIBLE SYSTEMS

In contrast, Axiomatic Design of large flexible systems provides a natural engineering design description of transportation systems. Axiomatic Design has been previously applied to transportation applications in the design of intersections [Pena *et al.*, 2010; Thompson *et al.*, 2009a; Thompson *et al.*, 2009b; Yi and Thompson, 2011], airport terminals [Pastor, 2011], and shipping companies & ports [Celik *et al.*, 2009; Kulak, 2005; Celik, 2009]. While relevant to these applications, this work expands the scope to include the entire transportation system network.

To this end, the Axiomatic Design of large flexible systems proves a useful design tool. Suh [2001] defines large flexible systems as systems with many functional requirements that not only evolve over time, but also can be fulfilled by one or more design parameters. In transportation systems, the set of functional requirements is taken as the set of transportation processes, **FR** = {Transportation Processes}. The definition of a transportation process is adapted from Farid [2008] where it was used in a production system application.

Definition 6. Transportation Process: A transportation-resource-independent process $p_\mu \in P$ that transports individuals between stations.

The set of design parameters is taken as the set of transportation resources **DP** = {Transportation Resources}. This definition is similarly adapted for application to transportation systems [Farid, 2008].

Definition 7. Transportation Resource: A vehicle $h \in H$ capable of realizing one or more non-null transportation processes such as a bus or train.

Once the high-level functional requirements and design parameters have been established, they may be simultaneously decomposed to establish full functional and physical hierarchies as part of a rigorous engineering design process. While this goal is not the objective of this paper, establishing the development in terms of the evolving high-level system architecture variables in both function and form grounds the methodology within the engineering design literature.

2.3 TRANSPORTATION DEGREES OF FREEDOM: AN ANALOGY

The concept of degrees of freedom as applied to large flexible systems originated with research in automated reconfigurable manufacturing systems in which an analogy between mechanical and production degrees of freedom was drawn [Farid, 2007, 2008; Farid and McFarlane, 2006a]. So as to make this paper's developments more intuitive, the analogy --this time for transportation systems --is redrawn.

Production system degrees of freedom arose from an analogy between mechanical and production systems that holds equally well for transportation systems [Farid and McFarlane, 2006a]. At the most basic level, a mechanical system is defined by its kinematics which is described by links and coordinates [Shabana, 1998]. Links make up the physical composition of a mechanical system. Similarly, transportation systems are composed of transportation resources. Coordinates are used to express the time-evolution of a continuous state which results in motion. However, an event-driven evolution of discrete states is more appropriate for reconfigurable transportation system architecture. Cassandras and LaFortune [1999] have previously drawn this analogy between coordinates for time-driven systems and events for event-driven systems. Finally, when analyzing multi-body mechanical systems, the number of coordinates is calculated based upon the number of combinations of dimensions and links less any applicable constraints [Shabana, 1998]. For example, a fully free three-link system has 18 degrees of freedom: 6 dimensions for each of the three links. The analogy suggests that transportation system degrees of freedom would come from the feasible combinations of transportation processes and their associated resources less applicable constraints. Finally, mechanical degrees of freedom are classified as either scleronomic, i.e. time-independent, or rheonomic, i.e. time-dependent [Shabana, 1998]. This suggests that event-driven systems' degrees of freedom would be scleronomic or rheonomic in relation to their sequence dependence.

3 TRANSPORTATION DEGREES OF FREEDOM

This section reframes previous work on production degrees of freedom [Farid, 2007, 2008; Farid and McFarlane, 2006a] into a transportation system context. First, a measure of scleronomic transportation degrees of freedom is developed as a measure of the sequence-independent capabilities of the transportation systems. Next, a measure of rheonomic transportation degrees of freedom is developed to address sequence-dependent capabilities. Along the way, a number of modeling simplifications are made to reflect transportation's relative simplicity in comparison to manufacturing. Additionally, matrix-based developments are introduced to replace scalar-based ones found in previous work.

3.1 SCLERONOMIC TRANSPORTATION DEGREES OF FREEDOM

The scleronomic transportation degrees of freedom arise from the Axiomatic Design knowledge base for large flexible systems [Farid, 2008]. Its development is recounted here for clarity.

Suh uses the large flexible system design equation notation:

$$FR_1 \$ (DP_1, DP_2, DP_3)$$
$$FR_2 \$ (DP_2, DP_3) \qquad [3]$$
$$FR_3 \$ (DP_3)$$

to signify that FR_1 can be realized by design parameters DP_1, DP_2, or DP_3 [Suh, 2001]. Previous work reinterprets the design equation in Equation 3 in terms of a matrix equation using a Boolean knowledge base matrix J which contains the systems degrees of freedom [Farid, 2008].

$$FR = J \odot DP \qquad [4]$$

where matrix Boolean multiplication $C = A \odot B$ is equivalent to $C(i,k) = V_j A(i,j) \wedge B(j,k)$ where $V_j a_j = a_1 \vee a_2 \ldots a_{n-1} \vee a_n$ is the array-OR operation [Warshall, 1962; Farid, 2008].

The transportation system knowledge base found in Equation 4 describes the transportation system's capabilities and is defined formally as follows. A transportation system is composed of a set of transportation processes $P = \{p_1, \ldots p_{\sigma(P)}\}$ that transport passengers from an arbitrary station b_{y1} to b_{y2}. If B is taken as the set of stations, then by definition there are $\sigma^2(B)$ such processes. Of these, $\sigma(B)$ are "null" processes where no motion occurs. The rest of the paper adopts the indices convention that a transportation process p_u transports passengers from station b_{y1} to b_{y2} such that

$$u = \sigma(B)(y_1 - 1) + y_2 \qquad [5]$$

These transportation processes are realized by a set of resources $R = \{r_1, \ldots, r_{\sigma(R)}\}$ which realize them. An event $\varepsilon_{uv} \in E$ (in the discrete event system sense) [Cassandras and Lafortune, 1999] can be defined for each feasible combination of production process p_u being realized by resource r_v.

Definition 8. Transportation System Knowledge base: A binary matrix: J_S, of size $\sigma(P) \times \sigma(R)$ is defined where element $J_S(u,v) \in \{0,1\}$ is equal to one when event e_{uv} exists..

Interestingly, the Axiomatic Design knowledge base itself forms a bipartite graph [van Steen, 2010] between the set of processes (e.g. functional requirements) and resources (e.g. design parameters).

Proceeding with the development, a number of discrete holonomic constraints can apply in the operational time frame so as to eliminate events from the event set. These constraints are said to be *scleronomic* as they are independent of event sequence. Such constraints can arise from any phenomenon that reduces the capabilities of a transportation system e.g. vehicle breakdowns, line closures, or road detours. The description of the discrete holonomic constraints can be captured succinctly in a single binary matrix.

Definition 9. Transportation System Scleronomic Constraints Matrix: A matrix K_S of size $\sigma(P) \times \sigma(R)$ whose elements $K_S(u,v) \in \{0,1\}$ are equal to one when a constraint eliminates event e_{uv} from the event set.

So as to not exaggerate the transportation system capabilities with null processes of remaining at the same

station, these events are eliminated by convention in the context of this paper.

$$K_S(u,v) = \begin{cases} 1 & \text{if } \mod((u-1),\sigma(B)) = \\ & (u-1)/\sigma(B) \\ 0 & \text{otherwise} \end{cases} \quad [6]\text{Or}$$

equivalently,

$$K_S = not[\mathbf{I}^{\sigma(B)V} \otimes \mathbf{1}^{\sigma(R)T}] \qquad [7]$$

where $\mathbf{I}^m$ is the identity matrix of size $m \times m$, $\mathbf{1}^m$ is the ones vector of length m, the A^V operation is shorthand for vectorization vec() commonly implemented in MATLAB with the (:) operator, and $\otimes$ is the Kronecker tensor product.

From these definitions of J_S and K_S, follows the definition of scleronomic transportation degrees of freedom.

Definition 10. Scleronomic Transportation Degrees of Freedom [Farid, 2007, 2008]: The set of independent transportation events E_S that completely defines the available transportation processes in a transportation system. Their number is given by:

$$DOF_S = \sigma(E_S) = \sum_u^{\sigma(P)}\sum_v^{\sigma(R)}[J_S \ominus K_S](u,v) \quad [8]$$

where the $A \ominus B$ operation is "boolean subtraction" Alternatively, $A \ominus B$ is equivalent to $A \cdot \bar{B}$. Note that the boolean "AND" $\cdot$ is equivalent to the hadamard product, and $\bar{B} = not(B)$. In matrix form, Equation 8 can be rewritten in terms of the Frobenius inner product [Abadir and Magnus, 2005]:

$$DOF_S = \langle J_S, \bar{K}_S \rangle_F = tr(J_S^T \bar{K}_S) \qquad [9]$$

The form Equation 9 interestingly matches the form of the expression used for mechanical degrees of freedom. Furthermore, it allows the usage of the Axiomatic Design knowledge base for further detailed engineering design. Finally, the constraints matrix captures the potential for operational constraints like vehicle breakdowns, line closures, or road detours. As such, it allows a flexible expression of transportation system capabilities in the design and operational phases.

3.2 RHEONOMIC TRANSPORTATION DEGREES OF FREEDOM

The previous subsection recalled the development of transportation degrees of freedom as independent. However, a transportation system has constraints that introduce dependencies in the sequence of events. Rheonomic transportation degrees of freedom provide a sequence-dependent measure of the capabilities in a transportation system [Farid, 2008].

Definition 11. Rheonomic Transportation Degrees of Freedom [Farid, 2007, 2008]: The set of independent transportation strings Z that completely describes the transportation system language.

In other words, the transportation system language L can be described equally well in terms of the Kleene closure [Cassandras and Lafortune, 1999] of the scleronomic and rheonomic transportation degrees of freedom.

$$L = E^* = Z^* \qquad [10]$$

For mathematical tractability, the length of strings z is limited to two. Strings of longer length are discussed in Section 4.

Given string $z_{\xi\psi} = e_{u_1v_1}e_{u_2v_2} \in Z$, $\xi = \sigma(P)(u_1-1)+u_2$ and $\psi = \sigma(R)(v_1-1)+v_2$, $\forall$ $u_1,u_2 \in \{1,\sigma(P)\}$ and $\forall v_1,v_2 \in \{1,\sigma(R)\}$. Intuitively speaking, certain transportation events can follow one another, while others are not possible. These feasible strings can be captured succinctly in a single binary matrix J_ρ of size $\sigma^2(P) \times \sigma^2(R)$ whose elements $J(\xi,\psi) \in \{0,1\}$ are equal to one when string $z_{\xi\psi}$ exists and can be calculated as:

$$J_\rho = J_S \otimes J_S \qquad [11]\text{Allowing the}$$

presence of scleronomic constraints, Equation 11 becomes

$$J_\rho = [J_S \ominus K_S] \otimes [J_S \ominus K_S] \qquad [12]$$

As before, a binary constraints matrix K_ρ of size $\sigma^2(P) \times \sigma^2(R)$ is used to describe the potential elimination of strings $z_{\xi\psi} = e_{u_1v_1}e_{u_2v_2}$ from the transportation system string set. While K_S can equal zero, K_ρ has perpetual non-zero continuity constraints. In other words, in order for one degree of freedom to follow another, the destination of the former must be equivalent to the origin of the latter. Formally, the convention described in Equation 5 implies that u equals the sequence of digits $((y_1-1),y_2)$ in base $\sigma(B)$. This yields two useful results: $y_1 = (u-1)/\sigma(B)+1$ where '/' represents integer division and $y_2 = \mod((u-1),\sigma(B))+1$ where $\mod(x,y)$ represents the modulus of x with respect to y. Calculation of K_ρ is executed from a binary square constraint matrix C_ρ of size $\sigma(P) \times \sigma(P)$ which is defined as

$$C_\rho(u_1,u_2) = \begin{cases} 0 & \text{if } \mod((u_1-1),\sigma(B)) = \\ & (u_2-1)/\sigma(B) \\ 1 & \text{otherwise} \end{cases} \quad [13]$$

which may be more simply calculated in terms of the following matrix equation

$$C_\rho = \mathbf{1}^{\sigma(B)} \otimes [\mathbf{I}^{\sigma(B)} \otimes \mathbf{1}^{\sigma(B)T}] \qquad [14]$$

From this, the rheonomic transportation constraint matrix can be calculated

$$K_\rho = C_\rho^V \otimes [1^{\sigma^2(R)}]^T \qquad [15]$$

It follows that the number of rheonomic transportation degrees of freedom is:

$$DOF_\rho = \sum_\vartheta^{\sigma^2(P)}\sum_\psi^{\sigma^2(R)}[J_\rho \ominus K_\rho](\vartheta,\psi)$$
$$= \langle J_\rho, \bar{K}_\rho \rangle_F = tr(J_\rho^T \bar{K}_\rho) \qquad [16]$$

Equation 16 can be rewritten in a number of equivalent forms [Farid, 2013].

$$DOF_\rho = \sum_{u_1}^{\sigma(P)}\sum_{u_2}^{\sigma(P)}\sum_{v_1}^{\sigma(R)}\sum_{v_2}^{\sigma(R)}\left[[J_S \cdot \bar{K}_S](u_1,v_1)\right.$$
$$\left. \cdot [J_S \cdot \bar{K}_S](u_2,v_2) \cdot \bar{C}_\rho(u_1,u_2)\right] \qquad [17]$$

Equation [17] views rheonomic degrees of freedom as a sequence of binary conditions. An alternative approach is to rearrange the vector spaces such that.

$$J_R = [J_S \cdot \bar{K}_S]^V [J_S \cdot \bar{K}_S]^{VT}$$
$$K_R = C_\rho \otimes [\mathbf{1}_{\sigma(R)} \mathbf{1}_{\sigma(R)}^T]$$
[18]

Here, scleronomic transportation degrees of freedom are treated as a basis vector – as would typically be done with mechanical degrees of freedom. J_R strongly resembles an adjacency matrix where are the degrees of freedom are treated as nodes and are mutually connected. K_R consequently applies the perpetual rheonomic constraints. The rheonomic transportation degrees of freedom measure becomes

$$DOF_\rho = \sum_{w_1}^{\sigma(E_s)} \sum_{w_2}^{\sigma(E_s)} [J_R \ominus K_R](w_1, w_2)$$
[19]
$$= \langle J_R, \bar{K}_R \rangle_F = tr(J_R^T \bar{K}_R)$$

where $w = \sigma(R)(u-1) + v$.

This section has reused the Axiomatic Design large flexible system knowledge base to introduce the concept of scleronomic and rheonomic transportation degrees of freedom. These measures are used in the next section to enumerate the number of passenger itineraries.

4 ENUMERATED ITINERARIES – PASSENGER DEGREES OF FREEDOM

As inspired by research in product degrees of freedom [Farid, 2008], the passenger degrees of freedom measure takes advantage of the efforts in the previous section to measure the number of ways that a passenger in the transportation system may be transported from a desired origin to a final destination (i.e. the number of possible itineraries). The derivation rests on three definitions:

Definition 12. Passenger Event: A scleronomic transportation degree of freedom that permits a passenger's transport along one leg of an itinerary from a desired origin y_1 to a desired destination y_n.

Definition 13. Passenger Itinerary: An n-string of passenger events that permit the passenger's transport from a desired origin y_1 to a desired destination y_n.

Definition 14. Passenger Degrees of Freedom (DOF_ρ): The number of passenger itinerary strings in the language L_ρ between a desired origin y_1 to a desired destination y_n.

From these definitions, a straightforward derivation of the passenger degrees of freedom is to sum the itineraries consisting of 1 leg, 2 legs, up to the number of n legs deemed impractical by the passenger.

$$DOF_p = \sum_i^n DOF_{pi}$$
(20)

The number of direct routes follows from Equation 9 considering that only the process $u = \sigma(B)(y_1-1) + y_2$ is desired.

$$DOF_{p1} = \langle e_u^T J_S, e_u^T \bar{K}_S \rangle_F = tr((e_u^T J_S)^T (e_u^T \bar{K}_S))$$ (21)

where e_n represents n^{th} the elementary basis vector of appropriate size.

The number of two-leg routes uses the rheonomic transportation degrees of freedom found in Equation [19] but

requires that the scleronomic constraints matrices be updated from their original formulation in Equation [7] to incorporate the desired origin y_1 to a desired destination y_n..

$$K_{Sy_1} = not\left[[e_{y1}^{\sigma(B)} \otimes \mathbf{1}^{\sigma(R)}] \otimes [\mathbf{1}^{\sigma(R)}]^T \right]$$
$$K_{Sy_2} = not\left[[\mathbf{1}^{\sigma(R)} \otimes e_{y2}^{\sigma(B)}] \otimes [\mathbf{1}^{\sigma(R)}]^T \right]$$
[22]

and that J_R be updated accordingly.

$$J_{Ry_1y_2} = [J_S \cdot \bar{K}_{Sy_1}]^V [J_S \cdot \bar{K}_{Sy_2}]^{VT}$$
[23]

here e_n represents n^{th} the elementary basis vector of appropriate size.

The number of n-leg passenger routes is derived from the scalar form in Equation [17] where strings of the form $z = e_{u1v1} e_{u2v2}$ were considered. Extending this string to *n* events $z = e_{u1v1} \dots e_{unvn}$ yields the number of n-event rheonomic transportation degrees of freedom $DOF_{\rho n}$

$$= \sum_{u_1,\dots,u_n}^{\sigma(P)} \sum_{v_1,\dots,v_n}^{\sigma(R)} \left[\prod_x^{n-1} [J_S \cdot \bar{K}_S](u_x, v_x) \cdot \bar{C}_\rho(u_x, u_{x+1}) \right]$$
[24]
$$\cdot [J_S \cdot \bar{K}_S](u_n, v_n)$$

This rather cumbersome scalar form based upon single events can be simplified by recalling that the product in Equation [19] is a square adjacency matrix A_R between scleronomic transportation degrees of freedom.

$$A_R = J_R \cdot \bar{K}_R$$
[25]

Following the initial introduction to graph theory, where the nth power of an adjacency can be used to calculate the n-step paths through a network,

$$DOF_{\rho n} = \sum_{w_1}^{\sigma(E_s)} \sum_{w_2}^{\sigma(E_s)} A_R^{n-1}(w_1, w_2)$$
[26]

Here, the (n-1) power originates from the differences between the traditional formulation of the transportation network graph and that the Axiomatic Design based approach. To fix the passenger itineraries specifically from the desired origin y_1 to a desired destination y_n, Equation [26] becomes

$$DOF_{\rho n} = \sum_{w_1}^{\sigma(E_s)} \sum_{w_2}^{\sigma(E_s)} \left[A_{Ry1} A_R^{n-3} A_{Ry2} \right](w_1, w_2)$$ [27]

where

$$A_{Ry_1} = \left[[J_S \cdot \bar{K}_{Sy_1}]^V [J_S \cdot \bar{K}_S]^{VT} \right] \cdot \bar{K}_R$$
$$A_{Ry_2} = \left[[J_S \cdot \bar{K}_S]^V [J_S \cdot \bar{K}_{Sy_2}]^{VT} \right] \cdot \bar{K}_R$$
[28]

In this section, passengers were modeled in terms of sequences, which allowed for the enumeration of their itineraries in a measure called passenger degree of freedom. All measures continued to exhibit the same three common elements found in mechanical degrees of freedom: discrete events captured in Axiomatic Design knowledge bases, constraint matrices, and a Boolean difference of these two matrices.

The transportation degrees of freedom broadly measure "reconfiguration potential". The scleronomic transportation degree of freedom measures provide a quantitative description

of which transportation capabilities exist in the system and potentially how they can be changed. Mathematically, it can be described as a reconfiguration process **R** :

$$\mathbf{R}(J_S, K_S) \rightarrow (J_S', K_S') \qquad [29]$$

The rheonomic transportation degree of freedom measures provide a quantitative description of how transportation capabilities can be combined into sequences. In either case, these measures describe the impact of the desired set of reconfigurations on the system capabilities. Mathematically, it can be described by the transformation:

$$\mathbf{R}(J_R, K_R) \rightarrow (J_R', K_R') \qquad [30]$$

While the Axiomatic Design approach to the calculation is admittedly more complex than the traditional graph theoretic method, the Axiomatic Design approach explicitly represents the transportation system knowledge base and constraint matrices. Therefore, these matrices can be readily decomposed and incorporated into design processes specifically aimed to achieve system resilience and reconfigurability. Furthermore, active control solutions can be developed to evolve these matrices in the operational time scale.

5 ILLUSTRATIVE EXAMPLE: MEXICO CITY PUBLIC TRANSPORTATION SYSTEM

To demonstrate the application of the passenger DOF measures, the Mexico City Public Transportation System is taken as an illustrative example. This system is one of the largest of its kind in the world and includes various modes of transportation, such as light rail, the bus network, the Metro and Metrobus. It serves a population of approximately 25 million and has over 300 stations [Hewlett Foundation, 2012].

For the purpose of this example, the system boundary is narrowed down to a few square blocks around the City Center (Centro), which is considered the exact geographic center and hub of activity of most typical Mexican cities. This is done for two reasons: first, to simplify the analysis and ensure a better understanding of the developed degree of freedom measures; and second, because the DOF approach for reconfigurable transportation system development is extensible to systems of any size.

The system has a total of 9 public transportation system stations that fall within the defined system boundary (B). These include stops along Metro and Metrobus lines, excluding other modes of public transportation available in the city such as buses (no longer running in the city center) and light rail (mostly used to serve outlying areas to the north and south of the city that are not covered by other transportation modes). There are 2 considered modes of transport (H), the Metro and Metrobus, and 49 transportation processes.

The knowledge base for the system being analyzed is an 81×2 binary matrix J_S on a 1-hour time scale, where the rows represent possible transportation processes between stations and the columns represent the transportation resources (or modes: Metro and Metrobus). By definition, the transportation process is valid (1) if there exists at least one resource that can do the process within the allotted timeframe.

Its corresponding constraints matrix, K_S, is calculated from Equations 6 and 7.

A DOF_S of 56 represents the number of transportation processes that are possible within the 1-hour time scale with the two resources provided. DOF_ρ is the number of sequences of two processes that are possible in the same system. Aside from these values, it is interesting to note that the sum of the non-zero elements in each column serves as a measure of the flexibility of the given transportation mode; the sum of the elements in each row provide a measure of redundancy.

6 DISCUSSION: RECONFIGURABILITY APPLICATIONS IN TRANSPORTATION SYSTEMS

Axiomatic Design has proven a powerful tool to measure transportation degrees of freedom as a measure of reconfiguration potential. This section discusses three classes of applications for these developments: reconfigurable operations, reconfigurable planning, and reconfigurability valuation.

6.1 RECONFIGURABLE OPERATIONS

The concept of transportation degrees of freedom can be applied to achieve reconfigurable transportation system operations when the knowledge base and constraint matrices are taken over a short but regular time interval i.e. one hour. In such a case, a reconfiguration process can be said to occur from one hour to the next. For example, not all bus and metro lines are in service at all times in the day. Their periods of non-operation can be captured within the constraints matrix.

These observations suggest that there exist many types of constraints that limit the reconfiguration potential of the transportation system. One can easily conceive code that pushes trains without choice down a dedicated line. The resulting transportation system language would be $L = e_{\eta 1 r1} e_{\eta 2 r1} e_{\eta 3 r1}$ when it could have been written to support the language $L = z^*_{\zeta \psi 1}$. In essence, railway operators that engage in active real-time switching sequences can be viewed as making real-time reconfigurations, or eliminating scleronomic and rheonomic constraints all together. Fixed public transportation system schedules are another example of inflexible operations. Buses and trains leave at a fixed time from a fixed location irrespective of existing traffic conditions or vehicle breakdowns elsewhere in the system. Real-time transportation scheduling algorithms represent a key enabling technology for reconfigurable operations in the face of disturbances and shocks to the system.

6.2 PLANNING

The concept of transportation degrees of freedom can also be applied to long-term planning decisions. In the medium term, the schedules generated by transportation system operators represent a planning activity of which transportation system resources are going to be used to realize which transportation system processes. In the long term, the expansion of a transportation system network represents an expansion of the system knowledge base to include new transportation processes (i.e. rows in the knowledge base) and

new transportation resources/modes (i.e. columns in the knowledge base).

Returning to the Mexico City system as an example, the reader is taken back to the late 1990's, before the Metrobus was developed for Mexico City. Back then, typical city buses covered the streets of the downtown area, contributing to what was already the most heavily-congested traffic area in the city. Even worse, the service was lackluster due to the long trip times between locations that were oftentimes reached faster on foot rather than by taking a bus. The Metro, known then for being crowded to the point of being uncomfortable and a safety hazard, was avoided by many passengers. A decision was taken to expand the transportation system. Table 1 shows the degrees of freedom for the system before 2005 (8 stations) and the same values for the current system (9 stations, already shown in Section 5). The system flexibility and reconfigurability are shown to increase dramatically with the introduction of the Metrobus. Additionally, this new transportation mode runs mostly on surface streets on dedicated median lanes -which translates to virtually no traffic congestion.

Table 1. Mexico City Public Transportation System Degrees of Freedom.

Variable	Before 2005	After 2005
DOF_S	40	56
DOF	228	422

6.3 THE VALUE OF RECONFIGURABILITY

The concept of transportation degrees of freedom as a measure of reconfiguration potential draws questions about the value of this reconfigurability. To this end, it is important to recognize that each transportation degree of freedom can be associated with tangible measures that figure in ROI and cost/benefit decisions. In operations, each degree of freedom is associated with a passenger capacity and hence a revenue. Alternatively, it can be associated with a time of execution, energy consumption, greenhouse gas emissions, operating costs, and externalities. Furthermore, one can measure the ease of a reconfiguration process and value it in terms of time or monetary cost [Farid, 2007]. In such a strategy, it becomes possible to value reconfigurability as an operations-stage life cycle property. In planning decisions, each degree of freedom can be associated with not just an expected capacity and revenue, but also the required investment to make the degree of freedom possible. Similarly, such an approach can be used to model future energy consumption and greenhouse gas emissions from a perspective of technical planning rather than macroeconomic development.

7 CONCLUSIONS AND FUTURE WORK

This paper has developed a set of system measures called passenger degrees of freedom. The work rests firmly on the foundation of previous work in the field of reconfigurability measurement. Specifically, an analogy between mechanical and transportation degrees of freedom was drawn. The application-neutral Axiomatic Design model of a knowledge base of functional requirements and design parameters was contextualized to transportation processes and resources [Farid, 2007, 2008; Farid and McFarlane, 2006a] in an intuitive fashion.

The developed passenger degree of freedom measures came in two varieties. The scleronomic degrees of freedom assess available transportation processes irrespective of sequence. Second, the rheonomic degrees of freedom describe the independent paths available from one point to another. These measures were discussed both practically and theoretically. For the former, the measures provided an intuitive description of how the reconfiguration potential of the Mexico City public transportation network changed in the face of additional resources. It also represented potential reconfigurations in which stations and resources and the processes that they realize are added, modified or removed. These measures showed that in large flexible systems -such as transportation networks -many insights into the system structure can be gained if the allocation of pairs of processes was considered in relation to pairs of resources. In such a way, the measures gave a thorough understanding of the potential for reconfiguration.

From a theoretical perspective, the Axiomatic Design models have multiple advantages. Each of the measures developed form an absolute scale; thus facilitating measurement and quantitative comparison [Ejiogu, 1991; Kriz, 1988; Roberts, 1979; Stevens, 1946; Zuse, 1991] involving all forms of statistics including means and percentages. The measures provide a high level of objectivity and consistency that may allow them to be a potentially expressive tool in the evaluation of transportation systems. Lastly, the measures provide a significant amount of design feedback. Their functional form shows clearly that reconfiguration potential can be improved with additional resources and processes, and with careful attention to the emergence of system constraints.

From a modeling point of view, the Axiomatic Design models avoid any needless complicating detail. In a formal sense, every element in the knowledge bases is required for the associated degree of freedom measures. In an empirical sense, each element corresponds to a physical relationship fundamental to the desired reconfiguration.

In future work, the authors seek to extend the development of passenger degrees of freedom as part of a systematic approach to reconfigurability measures described elsewhere [Farid, 2007, 2008]. The measurement of "reconfiguration potential" only addresses half of the reconfigurability measurement question [Farid, 2008]. Further measures of "reconfiguration ease" are forthcoming [Farid, 2007, 2008; Farid and McFarlane, 2006b, 2007]. The integration of these two branches of reconfigurability research into more complex measures of key characteristics such as integrability and convertibility also provide a challenging avenue of future work [Farid, 2007]. Finally, all of these measures would benefit from their application into industrial case studies.

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