

A Hybrid Dynamic System Model for the Assessment of Transportation Electrification

Asha Viswanath

Engineering Systems and Management
Masdar Institute of Science and Technology
Abu Dhabi, United Arab Emirates
Email: asha.viswanath@gmail.com

Amro M. Farid

Engineering Systems and Management
Masdar Institute of Science and Technology
Abu Dhabi, United Arab Emirates
MIT Technology Development Program
Cambridge, MA USA
Email: afarid@masdar.ac.ae, amfarid@mit.edu

Abstract—In recent years, the electrification of transport has emerged as a trend to support energy efficiency and CO₂ emissions reduction targets. The true success, however, of this trend depends on the successful integration of electric vehicles on the infrastructure systems that support them. Left unmanaged electric vehicles may suffer from delays due to charging or cause destabilizing charging loads on the electrical grid. While many works have sought to mitigate these effects with advanced functionality such as coordinated charging, vehicle-to-grid stabilization, and charging queue management, few works have assessed these impacts as a holistic transportation-electricity nexus. Furthermore, rigorous dynamic system mathematical models that couple the kinematic and electrical states have yet to be developed holistically. This paper develops such a model in the hopes of its application by EV fleet operators to not just assess but also improve their operations & control. The hybrid dynamic system model is composed of a marked petri-net model superimposed on the continuous time kinematic and electrical state evolution. The application of the model is demonstrated on an illustrative example of modest size and sufficient functional heterogeneity.

I. INTRODUCTION

In recent years, the electrification of transport has emerged [1] as a trend to support energy efficiency and CO₂ emissions reduction targets [2]–[4]. Relative to their internal combustion vehicle (ICV) counterparts, electric vehicles (EV), be they trains or cars, have a greater “well-to-wheel” energy efficiency [3], [5]. They also have the added benefit of not emitting any carbon dioxide in operation and rather shift their emissions to the existing local fleet of power generation technology [6].

The true success of electric vehicles depends on their successful integration with the infrastructure systems that support them. From a transportation perspective, electric cars may only have a short range of 150km [7] but may still require 6–8 hours to charge [8]. From an electricity perspective, the charging loads can draw excessive power demands which may exceed transformer ratings, cause undesirable line congestion, or voltage deviations [9]. These loads may be further exacerbated temporally by similar charging patterns driven by similar work and travel lifestyles or geographically by the relative sparsity of charging infrastructure in high demand areas [10]. In effect, the electric vehicles and their supporting charging infrastructure couples the transportation and electrical systems

into a nexus.

Definition 1. Transportation-Electricity Nexus (TEN): A system-of-systems composed of a system with the artifacts necessary to describe at least one mode of transport united with an interdependent system composed of the artifacts necessary to generate, transmit, distribute and consume electricity.

As a result, the performance in the transportation domain can not be studied independently of the performance in the electrical domain. Furthermore, efforts to operate and control the performance in either domain requires an assessment model whose scope includes the functionality of both systems. Consider an EV taxi or bus operator. It may wish to implement a coordinated charging scheme [11]–[17] so as to limit the charging loads on the electrical grid. The local electric utility may even incentivize this EV taxi operator to implement a “vehicle-to-grid” scheme [18]–[20] to stabilize variability in grid conditions. The degree to which these coordination functions can be implemented ultimately depends on availability of vehicles at the charging infrastructure. This availability depends on the EV operator’s client demand and dispatching approach; which in turn is likely to depend on the local geography and traffic conditions.

Thus far, two works have been able to assess the coupling between the kinematic and the electrical state in a TEN. A simplified study based on the city of Berlin has been implemented on Matsim [20], while a full scale study has been completed for the city of Abu Dhabi [10], [21]–[24] using the Clean Mobility Simulator [25]. The latter of these introduced a transportation system performance measure called “quality of service” [10] to address the availability concerns expressed in EV adoption public attitude surveys. It also introduced power system line and bus safety criteria on the basis of IEEE reliability standards [10]. While simulating these performance measures is of great value, ultimately rigorous dynamic models are required that facilitate the real-time control and management of such measures.

This paper presents a hybrid dynamic model (HDM) of a road transportation-electricity nexus. The model utilizes marked petri-nets to track the discrete state of road vehicles while continuous differential equations describe the time evo-

lution of the kinematic and electrical states. The paper proceeds as follows. Section II provides the necessary background from graph theory, axiomatic design for large flexible systems, and petri-net theory. Section III then develops the HDM first as a more basic stochastic petri-net and then later introducing the continuous-time dynamics. Section IV then demonstrates the model in a simple illustrative example. Section V concludes the work with the practical implications on the operations and control of EV fleet operators.

II. BACKGROUND

This section provides the fundamental concepts upon which the hybrid dynamic model in Section III is developed. These originate from graph theory, axiomatic design for large flexible systems, and petri-nets. As these subjects draw from disparate disciplines, an introductory tone is taken for the potentially uninitiated reader.

A. Graph Theory

Graph theory is a long established field of mathematics with applications in many fields of science and engineering where artifacts are transported between physical locations [26]–[28]. A number of definitions are introduced for later use in the discussion.

Definition 2. Graph [27]: $G = \{B, E\}$ consists of a collection of nodes B and collection of edges E . Each edge $e \in E$ is said to join two nodes, which are called its end points. If e joins $b_1, b_2 \in B$, we write $e = \langle b_1, b_2 \rangle$. Nodes b_1 and b_2 in this case are said to be adjacent. Edge e is said to be incident with nodes b_1 and b_2 , respectively.

Definition 3. Bipartite Graph [27]: Graph G is bipartite if $B(G)$ can be partitioned into two disjoint subsets B_1 and B_2 such that each edge $e \in E(G)$ has one end point in B_1 and the other in B_2 : $E(G) \subseteq \{e = \langle b_1, b_2 \rangle | b_1 \in B_1, \text{ and } b_2 \in B_2\}$.

Definition 4. Directed graph (digraph) [27]: D , consists of a collection nodes B , and a collection of arcs E , for which we write $D = (B, E)$. Each arc $a = \langle b_1, b_2 \rangle$, is said to join node $b_1 \in V$ to another (not necessarily distinct) node b_2 . Vertex b_1 is called the tail of a , whereas b_2 is its head.

In transportation systems, the nodes often physically represent intersections or stations, while edges/arcs represent roads, rails or transportation routes.

One key methodological contribution of this work is its departure from the traditional graph theoretic definition described above. Despite its rich body of literature, traditional graph theory does not [29]:

- 1) Explicitly differentiate between heterogeneous modes of transport
- 2) Explicitly describe the system function, especially functions that do not involve a type of transportation.
- 3) Explicitly describe “null-processes” (i.e. the process of staying in the same location).
- 4) Facilitate further detailed engineering design in terms of system function and form.

- 5) Facilitate the description of faulted, or intentionally-offline functionality to support reconfigurable operation.

In the context of this work, these limitations are likely to impede the development of a dynamic model that spans both transportation and electrical functions.

B. Axiomatic Design for Large Flexible Transportation Systems

To overcome these limitations, Axiomatic Design for large flexible systems has been previously applied for both manufacturing [29]–[37]. Suh defines large flexible systems as systems with many functions that not only evolve over time, but also can be fulfilled by one or more design parameters [38]. In transportation systems, the set of functional requirements is taken as the set of transportation processes.

Definition 5. Transportation Process [30]: Given an arbitrary origin station b_{y_1} and an arbitrary destination b_{y_2} within a set of stations B , a transportation-resource-independent process $p_{t\tau} \in P_T$ transports individuals between b_{y_1} and b_{y_2} . A convention is adopted between the indices of stations and transportation processes such that:

$$\tau = \sigma(B)(y_1 - 1) + y_2 \quad (1)$$

where the $\sigma()$ gives the size of a set.

There are $\sigma^2(B)$ such transportation processes of which $\sigma(B)$ are “null processes” where no motion occurs. Note that these transportation processes are analogous to the *potential* edges between nodes and the self-loops in a traditional graph. Nevertheless, they are *formally* different because an edge in Axiomatic Design requires that an edge represent a feasible combination of process *and* resource.

The set of design parameters is taken as the set of transportation resources. Depending on the nature of the design problem, previous work has interpreted these resources as vehicles [30]–[32], stations, or even whole modes of transport [29], [33], [34].

The essential design activity of mapping function to form is then completed via a transportation system knowledge base.

Definition 6. Transportation System Knowledge Base [33]: Given a set of transportation processes P_T and a set of transportation resources R , an event $\epsilon_{\tau v} \in \mathcal{E}_T$ (in the discrete event system sense) [39] can be defined for each feasible combination of transportation process $p_{t\tau}$ being realized by resource r_v . The Transportation System Knowledge Base J_T is a binary matrix of size $\sigma(P_T) \times \sigma(R)$ where element $J_T(\tau, v) \in \{0, 1\}$ is equal to one when event $\epsilon_{\tau v}$ exists.

Interestingly, the transportation system knowledge base has an additional property in that it defines the transportation system’s scleronomic (i.e. sequence-independent) degrees of freedom [30].

Definition 7. Scleronomic Transportation Degrees of Freedom [33]: The set of independent transportation events \mathcal{E}_T that

completely defines the available transportation processes in a transportation system. Their number is given by:

$$DOF_T = \sigma(\mathcal{E}_T) = \sum_{\tau}^{\sigma(P_T)} \sum_v^{\sigma(R)} J_T(\tau, v) \quad (2)$$

The transportation system knowledge base has been further refined in systems where another function is being realized concurrently with the transportation process. In the example of a manufacturing system, material handlers must be designed to not just realize transportation processes for products but also holding processes for them as well [30].

Definition 8. Holding Process [30]: A material-handler and end-effector-independent process $p_{c\kappa}$ that holds raw material, work-in-progress, or final products during the transportation from one buffer to another.

In this case, a new knowledge base J_C of size $\sigma(P_C) \times \sigma(R)$ may be constructed to capture the holding capabilities of the production system. Then, the refined scleronomic transportation knowledge base J is formed using the kronecker tensor product and column selection [32].

$$J = \left[J_C \otimes \mathbf{1}^{\sigma(P_T)} \right] \cdot \left[\mathbf{1}^{\sigma(P_C)} \otimes J_H \right] \quad (3)$$

where \otimes is the Kronecker tensor product and $\mathbf{1}^n$ is a ones vector of length n .

The refined scleronomic transportation system degrees of freedom follows accordingly [32].

$$DOF_{TEN} = \sigma(\mathcal{E}) = \sum_{\omega}^{\sigma(P)} \sum_v^{\sigma(R)} J(\omega, v) \quad (4)$$

where it follows that the index of the refined scleronomic transportation degrees of freedom $\omega = \sigma(P_C)(\kappa - 1) + \tau$.

Ultimately, the Axiomatic Design for Large Flexible Systems approach is sufficiently rich to overcome the limitations described in the previous subsection. The next step is to apply this description of system structure to a dynamic model of system behavior.

C. Petri-Nets

Petri-nets offer a long established method for modeling and simulating the discrete-event dynamics of a system. Their usage is described by the following definitions.

Definition 9. Marked Petri Net (Graph) [40]: A bipartite directed graph represented as a 5-tuple $\mathcal{N} = (B, \mathcal{E}, A, W, Q)$ where:

- B is a finite set of places of size $\sigma(B)$.
- \mathcal{E} is a finite set of transitions/events of size $\sigma(\mathcal{E})$.
- $A \subseteq (B \times \mathcal{E}) \cup (\mathcal{E} \times B)$ is a set of arcs of size $\sigma(A)$ from places to transitions and from transitions to places in the graph.
- $W : A \rightarrow \{0, 1\}$ is the weighting function on arcs.
- Q is a marking (or discrete state) vector of size $\sigma(B) \times 1 \in \mathbb{N}^{\sigma(B)}$.

Definition 10. Petri Net Incident Matrix [40]: An incidence matrix A of size $\sigma(B) \times \sigma(\mathcal{E})$ whose elements:

$$A(y, j) = w(b_y, \epsilon_{\omega v}) - w(\epsilon_{\omega v}, b_y) \quad (5)$$

where j is a unique index mapped from the ordered pair (ω, v) .

Definition 11. Petri Net (Discrete-Event) Dynamics [40]: Given a binary firing vector U_{Dk} of size $\sigma(\mathcal{E}) \times 1$ and a petri-net incidence matrix A of size $\sigma(B) \times \sigma(\mathcal{E})$, the evolution of the marking vector Q is given by the state transition function $\Phi(Q, U_{Dk})$:

$$Q[k + 1] = \Phi(Q, U_{Dk}) = Q[k] + AU_{Dk} \quad (6)$$

While marked petri-nets are sufficient for discrete-event dynamics, at times, it is necessary to associate a timing with each event as it occurs. This timing may be deterministic or stochastic with the latter defined as follows.

Definition 12. Stochastic Petri Net (Graph) [41]: A 6-tuple $\mathcal{S} = (B, \mathcal{E}, A, W, Q, \Lambda)$ where $(B, \mathcal{E}, A, W, Q)$ is a marked petri net and Λ is a vector of size $\sigma(\mathcal{E}) \times 1 \in \mathbb{R}^{\sigma(\mathcal{E})+}$ that represents transitions' firing rate. The firing delay ΔT_j of a given transition ϵ_j is taken as a negative exponential probability density function $f(t, \lambda_j) = \lambda e^{-\lambda_j t}$ where $t_j \geq 0$.

III. MODEL DEVELOPMENT

The development of a hybrid dynamic model for the assessment of transportation electrification gains its inspiration from hybrid automata [40]. However, rather than developing the model on the basis of automata where states are represented centrally, this work uses petri-nets so as to represent the system's state in a distributed fashion that more intuitively represents the flow of vehicles within a transportation system. The development proceeds in two parts. First, a relatively basic stochastic petri-net model (SPNM) is developed in Section III-A. This model captures the discrete-event behavior of the microscopic flow of traffic. Next, Section III-B refines the SPNM to develop a hybrid dynamic model (HDM) that includes the continuous-time dynamics of individual vehicles.

A. A Stochastic Petri-Net Model

This subsection takes as its goal the development of a stochastic petri-net model of an electrified road transportation nexus that conforms to Definition 12. To that end, the definitions from Section II are extensively used.

First, the transportation system structure must be defined in terms of a traditional digraph.

Definition 13. Road Transportation Network: a road transportation network is a digraph $D = (B, E)$ where B is a set of nodes that physically represent intersections and stations and E is a set of arcs that physically represent roads that connect these nodes.

Rather than continuing the development of the SPNM with a traditional graph incidence or adjacency matrix, instead a refined transportation system knowledge is generated so as to explicitly describe the TEN's heterogenous function. The set

of transportation processes follows from Definition 5 while the set of transportation resources is taken as $R = B \cup E$. The transportation system knowledge base consequently follows directly from Definition 6. Then a set of charging processes are introduced by analogy to replace the holding processes used in manufacturing systems.

Definition 14. Charging Process: A resource-independent process $p_c \in P_c$ that positively or negatively affects an electric vehicle's state of charge. These processes may draw or inject the required energy into the interdependent electricity grid.

In the context of this work, $P_C = \{p_{c1}, \dots, p_{c5}\}$ where

- p_{c1} – null charging does not change the electric vehicles state of charge
- p_{c2} – discharge the EV SOC to the electric vehicle's propulsion system
- p_{c3} – charge the EV SOC by wire
- p_{c4} – charge the EV SOC wirelessly
- p_{c5} – inject the EV SOC to the electrical network by wire

These processes may be further differentiated depending on the need for different rates of SOC change. Returning to the Axiomatic Design, these processes may be realized by the set of transportation resources R . Non-electrical roads and stations effectively implement p_1 and p_2 . Charging stations are those stations that are capable of p_3 . The recent advances in online electric vehicles (OLEV) [42]–[45] require roads that are capable of p_4 . Finally, the emerging literature on “vehicle-to-grid” technology [18]–[20] utilizes p_5 .

From the set of charging processes, the associated charging system knowledge base is defined.

Definition 15. Charging System Knowledge Base: Given a set of charging processes P_c and a set of transportation resources R , an event $\epsilon_{\kappa v} \in \mathcal{E}_C$ (in the discrete event system sense) [39] can be defined for each feasible combination of charging process $p_{c\kappa}$ being realized by resource r_v . The Charging System Knowledge Base J_C is a binary matrix of size $\sigma(P_C) \times \sigma(R)$ where element $J_C(\kappa, v) \in \{0, 1\}$ is equal to one when event $\epsilon_{\kappa v}$ exists.

The refined transportation system knowledge base given by Equation 3 now captures all the electrical as well as transportation capabilities in the TEN. It is important to note (for later derivation) that a given transportation resource r_v can only realize one non-null charging process.

Once, the combined charging and transportation event set has been identified, the model development proceeds to their inclusion in the SPNM.

Definition 16. Transportation Electricity Nexus Stochastic Petri Net: A stochastic petri net where

- B – the set of places represents the set of stations.
- E – the set of transitions is equivalent to the refined transportation degrees of freedom
- $A \subseteq (B \times \mathcal{E}) \cup (\mathcal{E} \times B)$ is a set of arcs of size $\sigma(A)$ from places to transitions and from transitions to places in the graph.

- $W : A \rightarrow \{0, 1\}$ is the weighting function on arcs. $w(p_y, t_{\omega v}) = 1$ and $w(p_y, t_{\omega v}) = 1$ iff $J(\omega, v) = 1$.
- Q is a marking (or discrete state) vector which represents the number of vehicles at or departing from a given station.
- Λ the transition firing rate that determines a transition's exponentially distributed firing delay. The associated firing delay ΔT_j is physically interpreted as the duration over which the transition is firing.

The petri-net's discrete event dynamics follow from Equation 6.

As described in the introduction, one essential reason for the development of the model is to assess the impact of charging on the electrical grid. To that effect, the k^{th} firing vector is represented by $\sigma(\mathcal{E})$ equivalent stochastic continuous-time functions.

$$U_{kj}(t) = \begin{cases} U_{Dk}(j) & t_k \leq t \leq t_k + \Delta T_j \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

The definition for a charging load function consequently follows:

Definition 17. Transportation Resource Charging Load Function: Given a resource r_v which is capable of realizing an event ϵ_j at a charging rate of α_j (kiloWatts), the resource's charging load function $f_c(v, t)$ is given by:

$$f_c(v, t) = \sum_k \alpha_j U_{kj}(t) \quad (8)$$

This subsection has presented a methodical approach to the development of a SPNM that describes the discrete dynamics of a transportation-electricity nexus. The main benefit of such a model is that it greatly facilitates the assessment of the TEN's holistic performance. On the transportation side, an extensive body of literature exists for studying SPNM as queuing systems [40]. Meanwhile, on the electrical side, the transportation resource charging load functions may be incorporated directly into power flow analysis studies [10], [21].

From an implementation perspective, the SPNM requires an initial marking vector $Q[k]$ that represents the initial locations of all the road vehicles. It would also require a set of firing vectors which state when and where each vehicle would move, stay, charge or discharge. Ultimately, these firing vectors are assumed to be partially or entirely uncontrollable. If they are taken as fixed inputs of typical driving and charging patterns, then they are uncontrollable. However, some vehicles may be under the control of a “supervisory-operator” which specifies the firing vectors in response to the monitored conditions of the transportation-electricity nexus.

B. A Hybrid Dynamic Model

While the SPNM has significant advantages, its stochastic nature can be refined and replaced with deterministic model. Choosing the right probability density function and estimating the associated firing rate vector Λ is a non-trivial endeavor.

For this reason, the SPNM is replaced by a hybrid dynamic model where the stochastic nature of the former is replaced by a set of continuous-time differential equations that describe the traffic evolution of each road vehicle.

The development of the hybrid dynamic model gains its inspiration from hybrid automata [39]. To that effect, it is defined as follows:

Definition 18. Transportation Electricity Nexus Hybrid Dynamic Model: A 10-tuple $\mathcal{H} = (B, \mathcal{E}, A, W, Q, \Phi, U, X, F, domain)$ where

- (B, E, A, W, Q) is the underlying marked petri net,
- Φ is the discrete state petri-net transition function in Definition 11,
- U is a binary vehicle firing matrix of size $\sigma(B) \times \sigma(H)$ whose element $U_k(y, h) = 1$ when the k^{th} firing moves a vehicle h from station y .
- $X = [x_1, \dots, x_{\sigma(H)}]$ is a continuous-time vector representing the kinematic and electric state of each vehicle in a fleet of size $\sigma(H)$.
- f is a vector field. $f: Q \times X \times U \rightarrow X$. It describes the continuous-time evolution of these vehicles.
- $domain$ is a set of invariant conditions [40] which associates a discrete state Q to an interval of X and U within which X and U must remain in order to also remain in the discrete state Q .

As defined above, the HDM may be classified as a microscopic traffic model because it resolves the continuous state of each vehicle. In contrast, the underlying marked petri-net model resolves the total number of vehicles at or leaving a given station. To make the continuous time and discrete event states consistent, it is necessary to replace the discrete firing vector U_{Dk} with the more descriptive vehicle firing matrix U_k . The former may be found from the latter by:

$$U_{Dk} = U_k \mathbf{1}^{\sigma(H)T} \quad (9)$$

The state x_h must represent the kinematic and electrical state of a given vehicle h . To that effect, $x_h = [z_h, \dot{z}_h, s_h]^T$ where

- z_h – is the distance of the vehicle along a road segment in relative coordinates
- \dot{z}_h – is the speed of the vehicle along the road segment
- s_h – is the vehicle’s state of charge.

The vector field f is implemented as a state space differential equation of the form:

$$\dot{X} = f(Q, X, U_k) \quad (10)$$

In free driving conditions, the dynamics of each vehicle become entirely uncoupled and the speed of the vehicle \dot{z}_h is set to a constant speed β_1 whether it is moving along a road or parked at a station. Additionally, the charging rate α_1 is sufficient to describe all five types of charging processes.

$$\begin{aligned} \dot{z}_h &= \beta_1 \\ \ddot{z}_h &= 0 \\ \dot{s} &= \alpha_1 \end{aligned} \quad (11)$$

However, under more normal driving conditions with some congestion a car following model [46], [47] is typically used.

$$\begin{aligned} \dot{z}_h &= f(z_h) \\ \ddot{z}_h &= \alpha \dot{z}_h^\beta (t) \frac{(\dot{z}_{h-1}(t-T) - \dot{z}_h(t-T))}{(z_{h-1}(t-T) - z_h(t-T))^\gamma} \\ \dot{s} &= \alpha \end{aligned} \quad (12)$$

where $\ddot{z}_h, \dot{z}_h, z_h$ are the acceleration, speed and position of the h^{th} vehicle which follows $h-1$ vehicle. $\alpha > 0$, β and γ are model parameters that control the proportionalities and T is reaction time [46], [47]. The values of model parameters are all set to unity for the example discussed later.

Finally, the *domain* describes a set of invariant conditions upon which a given discrete state remains valid. In the context of the HDM, these conditions are useful for constraining the vehicles distance along the road segment and its state of charge within limits. For example,

$$\begin{aligned} 0 &\leq z_h \leq D_v \\ 0 &\leq s_h \leq S_{hmax} \end{aligned} \quad (13)$$

where D_v may be the road length and S_{hmax} may be the vehicle’s battery capacity.

This section has described the hybrid dynamic model which will be illustrated with an example in the next section.

IV. ILLUSTRATIVE EXAMPLE

In this section, the TEN HDM is demonstrated with an illustrative example. Consider a transportation system composed of three mutually connected stations at the locations $(0,0)$, $(0,2)$ and $(1,1)$ kilometers. Of these transportation system resources, one road and one station is given a charging capability. Following the methodology previously described, the petri-net for the associated transportation-energy nexus is shown in Figure 1. For clarity, charging events/transitions are colored green while those associated with discharging are colored black.

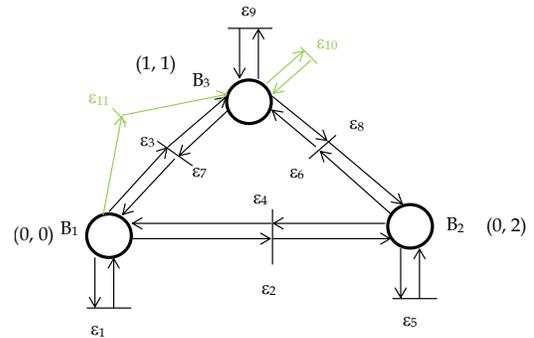


Figure 1: Petri-Net of the 3-Station TEN

The TEN also has five electric vehicle which will be transported and charged. As shown in Table I, the discrete state evolution is described by an itinerary for each vehicle. One essential point in this research is the *need* for full itineraries as input data rather than simply origin-destination data. This is in contrast to most microscopic traffic simulation packages

[48], [49] where input origin-destination pairs are the norm. Origin-destination pairs as input data fundamentally prevent the study of stationary vehicles that may be charging or idling in a charging queue. Using itineraries for input data relieves this problem because it becomes clear where and when a vehicle may be parked.

Table I: Electric Vehicle Itineraries

EV-ID	Itinerary
1	$B_1 \rightarrow B_2 \rightarrow B_1 \rightarrow B_3 \rightarrow B_2 \rightarrow B_1 \rightarrow B_2 \rightarrow B_3 \rightarrow B_1$
2	$B_1 \rightarrow B_2 \rightarrow B_3 \rightarrow B_1 \rightarrow B_2 \rightarrow B_1 \rightarrow B_3 \rightarrow B_1$
3	$B_1 \rightarrow B_3 \rightarrow B_2 \rightarrow B_1 \rightarrow B_3 \rightarrow B_1 \rightarrow B_2 \rightarrow B_3 \rightarrow B_1$
4	$B_2 \rightarrow B_3 \rightarrow B_1 \rightarrow B_2 \rightarrow B_1 \rightarrow B_3 \rightarrow B_2$
5	$B_2 \rightarrow B_1 \rightarrow B_2 \rightarrow B_3 \rightarrow B_1 \rightarrow B_2 \rightarrow B_1 \rightarrow B_3 \rightarrow B_1 \rightarrow B_2 \rightarrow B_3$

To describe the continuous time dynamics, the charging and discharging rates α_1 are set to 9 and -9%/minute. The initial velocity of all vehicle β_1 are set to 1km/minute.

The simulation of the TEN HBD proceeds with the translation of the itinerary data into a binary vehicle firing matrix U . Figure 2 depicts the firing matrix such that it is clear when each vehicle will experience each event and for how long.

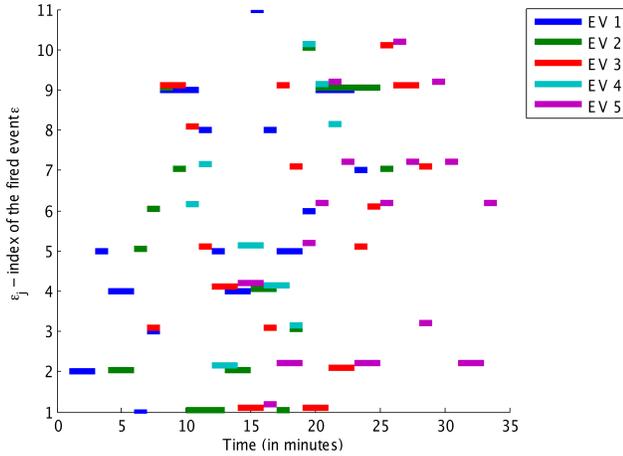


Figure 2: Graphical depiction of the binary firing matrix U . The duration of each firing event for each vehicle is shown.

Finally, the results of the continuous-time domain simulation are shown in Figure 3. The state variables z_h are resolved back to absolute (x, y) coordinates from the locations of the stations. The speed \dot{z}_h state variables show that generally speaking the electric vehicles experience free roadways; moving at 1km/minute for the majority of the simulation. The state of charge variables s_h , generally decrease as the vehicles move along the road. However, Vehicle 1 charges at $T=15\text{min}$ while still moving along Road 1-3. In contrast, Vehicle 4 and then later simultaneously Vehicles 3, and 5 charge at Station B_3 and are stationary during that time. The simultaneous charging appears as a charging load of double magnitude.

V. CONCLUSIONS

This paper has developed a hybrid dynamic model for the electrification of a transportation system. The model is

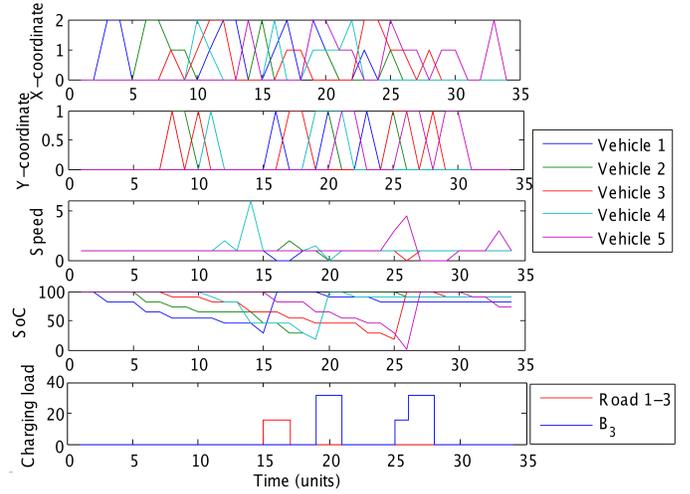


Figure 3: Continuous Time Domain Simulation of TEN HDM

comprised of a marked petri-net model to describe the discrete-state evolution of vehicles and a continuous-time car following model. The application of the model is demonstrated on an illustrative example of modest size and sufficient functional heterogeneity. The results demonstrate that the model is capable of resolving the kinematic state of the vehicle fleet while also keep track of each vehicle's state of charge. The subsequent charging load is treated as an output function which may be used directly in power system power flow analyses. In such a way, this hybrid dynamic model captures both the transportation as well as electrical functionality in a transportation-energy nexus. In future work, this model may be applied by an operator of electric vehicles and/or trains to coordinate route dispatch, charging queues, charging dispatch, and vehicle-2-grid stabilization simultaneously.

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