# Static Resilience of Large Flexible Engineering Systems: Part I – Axiomatic Design Model

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***Abstract*—Our modern life has grown to depend on many and nearly ubiquitous large complex engineering systems. In recent years, many disciplines have seemingly come to ask the same question: "In the face of assumed disruption, to what degree will these systems continue to perform and when will they be able to bounce back to normal operation". This paper seeks to partially fulfill this need with *static* resilience measures for *large flexible engineering systems* based upon an axiomatic design model. Given that the measurement of resilience is an indirect measurement process based upon models and formulaic measures, this two part paper is similarly organized. In Part I, the model is developed upon graph theory, axiomatic design for large flexible engineering systems (LFESs), and a tight analogy between mechanical systems and LFESs. Central to the development is the concept of structural degrees of freedom as the available combinations of systems processes and resources which individually describe system capabilities or sequentially give a sense of the skeleton of a system's behavior. In Part II, the structural degree of freedom model is used to enumerate the service paths through a LFES along which valuable artifacts flow. The work then compares the value and quantity of service paths before and after a disruption as measures of static resilience – or survivability. A full illustrative example from the production system domain is provided. It is followed by a thorough discussion of the proposed resilience measures relative to the recent literature.**

***Index Terms*—resilience, large complex systems, axiomatic design, graph theory, resilient systems, resilience measurement**

## I. Introduction

Our modern life has grown to depend on many and nearly ubiquitous large complex engineering systems [1]. Transportation, water distribution, electric power, natural gas, healthcare, manufacturing and food supply are but a few. These systems are characterized by an intricate web of interactions within themselves [2] but also between each other [3]. Our heavy reliance on these systems coupled with a growing recognition that disruptions and failures; be they natural or man-made; unintentional or malicious; are inevitable. Therefore, in recent years, many disciplines have seemingly come to ask the same question: "How *resilient* are these systems?" Said differently, in the face of assumed disruption, to what degree will these systems continue to perform and when will they be able to bounce back to normal operation [4]. Furthermore, the major disruptions of 9/11, the 2003 Northeastern Blackout, and Hurricane Katrina and Sandy has caused numerous agencies to make resilient engineering systems a central policy goal [5].

Naturally, a large body of academic literature has developed on the subject across multiple disciplines [5], [6]. One major conclusion, is that the field of resilience engineering is still emerging and requires formal quantitative definitions and frameworks [4], [5]. A key element to such rigorous approaches is the development of resilience measures which many, even recently, have identified as an area for concerted effort [4]–[10]. Such resilience measures would not only quantify resilience but could also inform designers and planners in advance how to best improve system resilience.

### A. Contribution

This paper seeks to partially fulfill this need with *static* resilience measures for *large flexible engineering systems* based upon an axiomatic design model [11]. Much of the resilience measurement literature divides the life cycle property into two complementary aspects: a static "survival" property which measures the degree of performance after a disruption, and a dynamic "recovery" property which measures how quickly the performance returns to normal operation [1], [4]–[7], [9], [10]. Additionally, many resilience measures in the literature depend on traditional graph theoretic applications. The choice of axiomatic design over (traditional) graph theory allows the paper's scope to expand from *homo-functional* to *hetero-functional* systems. The paper's contribution builds upon previous work in which axiomatic design was also applied to reconfigurable manufacturing and transportation systems [12]–[20]. One notable theme in the prior work was the enumeration of paths in these large flexible engineering systems which will be used here in the resilience measure development.

### B. Scope

This paper restricts its scope to large flexible engineering systems.

**Definition 1.** Large Flexible Engineering System (LFES) [11]: an engineering system with many functional requirements (i.e. system processes) that not only evolve over time, but also can be fulfilled by one or more design parameters (i.e. system resources).

The paper also addresses static resilience as a measure of the degree a system can continue to perform after disruption.

The dynamic "recovery" nature of resilience is left for future work.

### C. Paper Outline

The measurement of resilience is naturally an indirect measurement process [21]. Therefore, this two-part paper is guided by Figure 1. Part I develops the axiomatic design model. Part II develops the formulaic measures. In this paper, Section II orients the discussion in terms of the two foundations of the work: graph theory and axiomatic design. It also makes an analogy between mechanical systems and LFES upon which much of the work is built. Next, Section III introduces a model of "Structural Degree of Freedom" as a generalization of previous work [12]–[20] applied to production and transportation systems. Section III-E concludes the work and prepares the reader for Part II.

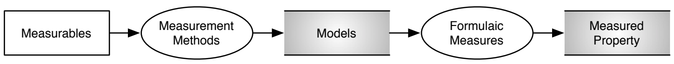


Fig. 1: A Generic Indirect Measurement Process [21]

## II. Background

This section summarizes the methodological foundations found in graph theory and axiomatic design in order to introduce the concept of "structural degrees of freedom" in the next section. Section II-A gives a brief introduction to graph theory while Section II-B introduces the application of axiomatic design to LFESs. Finally, Section II-C presents an analogy between mechanical systems and LFESs upon which the rest of the paper is based.

### A. Graph Theory Introduction

As mentioned in the introduction, much of the existing resilience measurement literature has been based on graph theory [22]–[24]. A number of definitions and theorems from this field are introduced for later in the development.

**Definition 2.** A graph [23]: $G = \{V, E\}$, consists of a collection of nodes $V$ and a collection of edges $E$. Each edge $e \in E$ is said to join two nodes which are called its end points. If $e$ joins $v_1, v_2 \in V$, we write $e = \langle v_1, v_2 \rangle$. Nodes $v_1$ and $v_2$, in this case, are said to be adjacent. Edge $e$ is said to be incident with nodes $v_1$ and $v_2$ respectively.

**Definition 3.** Adjacency matrix [23]: $A$, is binary and of size $\sigma(V) \times \sigma(V)$ and its elements are given by

$$A(i,j) = \begin{cases} 1 & \text{if } \langle v_i, v_j \rangle \text{ exists} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where the operator $\sigma()$ gives the size of a set.

**Theorem 1.** *Number of Paths in a Graph [22]: The number of n-step paths between nodes i and j is given by* $A^N(i,j)$.

**Theorem 2.** *Number of Loops in a Graph [22]: The number of n-step loops from node i back to itself is given by* $A^N(i,i)$.

**Definition 4.** Diameter of a Graph [22]: The length of the longest geodesic (i.e shortest) path $D$ between any nodes in a graph measured in number of steps.

While graph theory for decades has presented a useful abstraction across many applications, it has limitations from an engineering design and systems engineering perspective [25]. Traditionally speaking, graph theory has been applied to systems where artifacts are transported between physical locations. The main challenge is finding robust approaches to linking nodes and arcs to physical variables [22]. The above definitions focus on the abstract form of the system and less so its function. The system's functions themselves are not explicitly stated. Furthermore, because the system's functions and its realizing form have been abstracted away such approaches may not straightforwardly lend themselves to detailed engineering design of the system's component functions [26], [27]. This disconnect may impede rigorous approaches with which resilience can be engineered into the system. As a final observation, graph theory has been traditionally applied to large flexible *homo-functional* engineering systems.

### B. Axiomatic Design for Large Flexible Engineering Systems

In contrast, axiomatic design for LFES provides a natural engineering design methodology that overcomes many of these limitations. In the context of this work, the "functional requirements" and "design parameters" mentioned in Definition 1 are understood to be the system processes and resources respectively. At the highest level of abstraction, these resources $R = M \cup B \cup H$ may be classified into transforming resources $M = \{m_1 \ldots m_{\sigma(M)}\}$, independent buffers $B = \{b_1 \ldots b_{\sigma(B)}\}$, and transporting resources $H = \{h_1 \ldots h_{\sigma(H)}\}$ [12]–[17]. The set of buffers $B_S = M \cup B$ is also introduced for later simplicity. Similarly, the high level system processes are formally classified into their transformation and transportation varieties $P = P_\mu \cup P_\eta$ [12]–[17].

**Definition 5.** Transformation Process [12]–[17]: A resource-independent, technology-independent process $p_{\mu j} \in P_\mu = \{p_{\mu 1} \ldots p_{\mu \sigma(P_\mu)}\}$ that transforms an artifact from one form into another.

**Definition 6.** Transportation Process [12]–[20]: A resource-independent process $p_{\eta u} \in P_\eta = \{p_{\eta 1} \ldots p_{\eta \sigma(P_\eta)}\}$ that transports artifacts from one buffer $b_{sy_1}$ to $b_{sy_2}$. There are $\sigma^2(B_S)$ such processes of which $\sigma(B_S)$ are "null" processes where no motion occurs. Furthermore, the convention of indices $u = \sigma(B_S)(y_1 - 1) + y_2$ is adopted.

These high level systems processes are defined to include both underlying physical function as well as their supporting enterprise control activities [28].

**Example 1.** Table I describes the system processes and resources for four indicative LFESs.

The system processes and resources may be related to the system resources through the use of the axiomatic design equation for LFESs [13]–[19]

$$P = J_S \odot R \tag{2}$$

Table I: System Processes & Resources in LFESs

| | $P_\mu$ | $P_\eta$ | $M$ | $B$ | $H$ |
|---|---|---|---|---|---|
| **Production** | Trans-formation | Trans-portation | Value-Adding Machines | Buffers | Material Handlers |
| **Transportation** | Entry/Exit | Trans-portation | Stations | Stations | Vehicles |
| **Power Grids** | Generation/ Consump-tion | Transmission | Generators/ Loads | Storage | Lines |
| **Water** | Extract/ Treat/ Pollute/ Dispose | Distribute | Treatment/ Demands | Storage | Lines |

where $J_S$ is a binary matrix called a "knowledge base", and $\odot$ is "matrix boolean multiplication" [13]–[19]. In other words, the system knowledge base itself forms a bipartite graph [23] which maps the set of system processes to their resources. Also of interest, the system knowledge base is a concise description of LFES structure.

**Definition 7.** System Structure [29](page26): the parts of a system and the relationships amongst them. It is described int terms of

- A list of all components (i.e. resources) that comprise it.
- What portion of the total system behavior (i.e. processes) is carried out by each component (i.e. resources).
- How the components (i.e. resources) are interconnected.

### C. Mechanical Systems & LFES: An Analogy

This discussion on structural degrees of freedom in the next section is best motivated by an analogy between mechanical systems and LFESs. Previous works have loosely drawn this analogy [12]–[18]. This work further strengthens the analogy for clarity and intuition development.

*1) Kinematic Degrees of Freedom:* In mechanical systems, the number of *kinematic* degrees of freedom (or generalized coordinates) is commonly given by [30]

$$DOF = n_l * n_d - n_k \tag{3}$$

where $n_l$ is the number of links, $n_d = 6$ is the number of primitive coordinates, and $n_k$ is the number of applied scleronomic (i.e time independent) constraints that confine motion. If any individual constraint is assumed to affect only a single combination of link and coordinate, then Equation 3 can be written as [13]–[20]:

$$DOF = \sum_{i}^{n_d}\sum_{j}^{n_l}\left[J_S \ominus K_S\right](u,v) = \sum_{i}^{n_d}\sum_{j}^{n_l} A_S(u,v) \tag{4}$$

where $J_S$ is a binary ones matrix of size $n_d \times n_l$ whose elements describe the possible combinations of dimension $i$ with link $j$, $K_S$ is a constraints matrix of same size that eliminates these feasible combinations and $\ominus$ is element-wise boolean subtraction. $A \ominus B = A \cdot \bar{B}$. Note that the boolean "AND" $\cdot$ is equivalent to the hadamard product and $\bar{B} = \text{not}(B)$. The generalized coordinates maybe written in matrix form $\mathbf{x}(u,v)$. However, more often than not for mathematical convenience, they are vectorized to $\mathbf{x}^V$ where the notation $A^V$ is shorthand for $vec(A)$. It follows that the generalized coordinates $\mathbf{x}^V$ and their time derivatives $\dot{\mathbf{x}}^V$ can be explicitly written in terms of the primitive coordinates $\mathbf{x}_p^V$ and their time derivatives $\dot{\mathbf{x}}_p^V$ by:

$$\mathbf{x}^V = A_S^V \cdot \mathbf{x}_p^V \tag{5}$$

$$\dot{\mathbf{x}}^V = A_S^V \cdot \dot{\mathbf{x}}_p^V \tag{6}$$

The analogy [12]–[18] between mechanical systems and LFES is made such that

Mechanical System : LFES
kinematic dimension : system process
mechanical link : system resource
scleronomic constraint : scleronomic constraint

On this basis, the following section will show that the sequence-independent structural degrees of freedom of LFES follow Equation 4.

*2) Dynamic Relations between Degrees of Freedom:* The analogy between mechanical systems and LFESs continues when dynamic relations are included to derive the equations of motion. In principle, the mechanical system state $\mathbf{X} = [\mathbf{x}^V, \dot{\mathbf{x}}^V]^T$ has $2DOF$ variables. The time derivatives between $\mathbf{x}^V$ and $\dot{\mathbf{x}}^V$ impose $DOF$ equations. The dynamic relations, therefore, *must* impose another $DOF$ equations to determine the evolution of the system state. This is often done via the Lagrange equations [31]:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}(\mathbf{x}^V, \dot{\mathbf{x}}^V)}{\partial \dot{x}_{ij}}\right) - \frac{\partial \mathcal{L}}{\partial x_{ij}} = \Xi_{ij} \tag{7}$$

where $\Xi_{ij}$ is an external force acting along direction $i$ on link $j$ and the Lagrangian $\mathcal{L}(\mathbf{x}^V, \dot{\mathbf{x}}^V) = \mathcal{T}^* - \mathcal{V}$ is defined as the difference between the system's kinetic co-energy $\mathcal{T}^*$ and the system's potential energy $\mathcal{V}$.

Now consider the Lagrangian of a 3D "Bravais" lattice shown in Figure 2. It has uniform masses $m$ and springs of spring constant $k$ along the cardinal axes. A traditional derivation of the Lagrangian considering constant, linear and square terms gives [32]

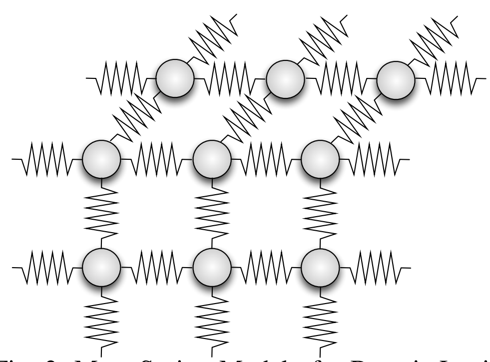

Fig. 2: Mass-Spring Model of a Bravais Lattice

$$\mathcal{L} = \frac{m}{2}\sum_{u}^{n_d=3}\sum_{v}^{n_l}\dot{x}_{uv}^2 - \frac{k}{2}\sum_{u_1}^{n_d=3}\sum_{u_2}^{n_d=3}\sum_{v_1}^{n_l}\sum_{v_2}^{n_l} \left[(x_{u_1v_1} - x_{u_2v_2})^2 \cdot A_c(v_1,v_2) \cdot A_g(j_1,j_2)\right] \tag{8}$$

Here, $A_c$ is the constitutive relations adjacency matrix that represents the graph consisting of masses as nodes and springs as edges. It applies the constitutive relations as constraints in the potential energy term. Similarly, $A_g$ is the geometric adjacency matrix that applies geometric relations which effectively state that each cardinal axis is independent from the other. $A_g = \mathbf{I}$. Equation 8 can be written explicitly in matrix form in terms of the primitive coordinates as:

$$\mathcal{L} = \frac{m}{2}\left[\dot{\mathbf{x}}_p^{VT} \cdot A_S^{VT}\right]\left[A_S^V \cdot \dot{\mathbf{x}}_p^V\right] + k\left[\mathbf{x}_p^{VT} \cdot A_S^{VT}\right] A_\rho \left[A_S^V \cdot \mathbf{x}_p^V\right] \quad (9)$$

where $A_\rho = A_g \otimes A_c$ where $\otimes$ is the Kronecker tensor product [33]. In this form, the Lagrangian explicitly shows that if a mass were to somehow be "kicked out" of the lattice, it would correspond to a change in $A_S$ affecting the system's kinetic co-energy and potential energy. Similarly, in this form, the Lagrangian explicitly shows that if a spring were to fail or to be added, it would correspond to a change in $A_\rho$ and correspondingly change the system's potential energy. Furthermore, Equations 3-9 can be easily generalized for physical systems of multiple energy domain [34], [35] while still retaining the presence of the $A_S$ and $A_\rho$ matrices. On this basis, the following section will show that the sequence-dependent structural degrees of freedom rely on $A_\rho$.

*3) Outputs of Interest:* The last part of the analogy between mechanical systems and LFESs is built on the recognition that sometimes some of the generalized coordinates have a "practical" importance. To that effect, it is not uncommon to use a measurement matrix $C$ to extract an output vector $\mathbf{Y}$ as a subset of the generalized coordinates.

$$\mathbf{Y} = \mathbf{CX} \quad (10)$$

On this basis, the analogy between mechanical systems and LFESs is further developed. Stated in its entirety, the analogy is:

$$\begin{aligned} \text{Mechanical System} &: \text{LFES} \quad (11)\\ \text{kinematic dimension} &: \text{system process}\\ \text{mechanical link} &: \text{system resource}\\ \text{scleronomic constraint} &: \text{scleronomic constraint}\\ \text{constitutive \& geometric relations} &: \text{rheonomic constraints}\\ \text{measurement matrix} &: \text{service selector matrix} \end{aligned}$$

## III. Structural Degrees of Freedom Model

This section introduces the structural degree of freedom models & measures as a generalization of previous work applied to production and transportation systems [12]–[20]. Given the abstract nature of the mathematical treatment, the interested reader is referred to the supporting references for illustrative examples and case studies. The discussion proceeds in four parts: 1.) Measurables & Measurement Methods, 2.) Sequence independent structural degrees of freedom, 3.) Sequence dependent structural degrees of freedom 4.) Service-specific structural degrees of freedom.

### A. Measurables & Measurement Methods

As shown in Figure 1, the measurement of resilience is naturally an indirect measurement process. It requires that measurables be directly measured with measurement methods and then placed into models from which formulaic measures can give the desired measurement property of resilience. In this work, the measurables are the system processes and resources. They may be measured directly by simply counting them once the measurer has determined a consistent ontological basis for defining them; especially for the transformation processes [14], [36]. Manual counting is naturally tedious for any truly large and geographically distributed LFES. Instead, this measurement process assumes that there exists a virtual model of the system as is typically found in their IT-based management systems. For the sake of universality and clarity, this paper discusses such a virtual model using SysML terminology. Figure 3, provides a SysML example of system processes, resources, and their allocation via swim lanes.

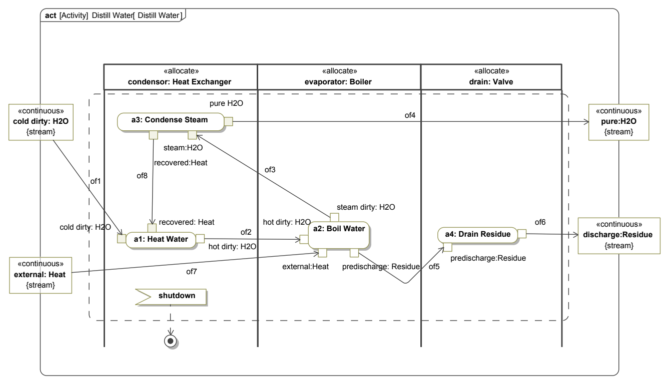


Fig. 3: Example: System Processes, Resources & Allocation

### B. Sequence-Independent Structural Degrees of Freedom

The heart of the structural degrees of freedom concept rests in the realization that an allocated action $e_{wv} \in E$ (in the SysML activity diagram sense) [37] can be defined for each feasible combination of system process $p_w$ and resource $r_v$. SysML classifies actions on the basis of whether they are *streaming* or *nonstreaming*. In the case of the former, the allocated action would be described by a continuous time differential equation [34], [35]. In the case of the later, the allocated action would be described as a discrete event [38]. With these notions in mind, reconfigurations and disruptions can add or remove these allocated actions or potentially reallocate a process to a resource.

**Definition 8.** LFES Knowledge Base [12]–[20]: A binary matrix $J_S$ of size $\sigma(P) \times \sigma(R)$ whose element $J_S(w, v) \in \{0, 1\}$ is equal to one when action $e_{wv}$ exists.

The development of structural degrees of freedom continues with the introduction of a number constraints as is found in mechanical degrees of freedom. Here, the constraints are discrete and can apply in the operational time frame so as to eliminate actions from the action set. These constraints are said to be *scleronomic* as they are independent of action sequence. Such constraints can arise from any phenomenon that reduces the capabilities of a LFES e.g. resource breakdowns, inflexibly implemented processes and their control.

**Definition 9.** LFES Scleronomic Constraints Matrix [12]–[20]: A binary matrix $K_S$ of size $\sigma(P) \times \sigma(R)$ whose element $K_S(w,v) \in \{0,1\}$ is equal to one when a constraint eliminates event $e_{wv}$ from the event set.

From these definitions of $J_S$ and $K_S$, follows the definition of LFES sequence-independent structural degrees of freedom.

**Definition 10.** LFES Sequence-Independent Structural Degrees of Freedom [12]–[20]: The set of independent actions $E_S$ that completely defines the available processes in a LFES. Their number is given by:

$$DOF_S = \sigma(E_S) = \sum_{w}^{\sigma(P)} \sum_{v}^{\sigma(R)} [J_S \ominus K_S](w,v) \tag{12}$$

$$= \sum_{w}^{\sigma(P)} \sum_{v}^{\sigma(R)} A_S(w,v) \tag{13}$$

In matrix form, Equation 12 can be rewritten in terms of the Frobenius inner product [33].

$$DOF_S = \langle J_S, \bar{K}_S \rangle_F = tr(J_S^T \bar{K}_S) \tag{14}$$

These LFES sequence-independent structural degrees of freedom may also be classified into their transformational ($DOF_M$) and transportational ($DOF_H$) variants as shown in Table II. In this case, it follows that [12]–[17]:

$$J_S = \left[ \begin{array}{c|c} J_M & \mathbf{0} \\ \hline \multicolumn{2}{c}{J_{\bar{H}}} \end{array} \right] \tag{15}$$

$$K_S = \left[ \begin{array}{c|c} K_M & \mathbf{0} \\ \hline \multicolumn{2}{c}{K_{\bar{H}}} \end{array} \right] \tag{16}$$

One notable difference is the measure for refined transportation degrees of freedom $DOF_{\bar{H}}$. Definition 6 for a transportation process fundamentally limits their number to $\sigma^2(B_S)$. However, it is often useful to distinguish between transportation processes on the basis of the holding processes of the associated resource.

Table II: Types of LFES Sequence-Independent Degree of Freedom Measures [13]–[17]

| Measure | Process Element | Resource Element | Knowledge Base | Constraint Matrix | Measure Function |
|---|---|---|---|---|---|
| $DOF_M$ | $p_{\mu j}$ | $m_k$ | $J_M$ | $K_M$ | $\langle J_M, \bar{K}_M \rangle_F$ |
| $DOF_H$ | $p_{\eta u}$ | $r_v$ | $J_H$ | $K_H$ | $\langle J_H, \bar{K}_H \rangle_F$ |
| $DOF_{\bar{H}}$ | $P_{\gamma \varrho}$ | $r_v$ | $J_{\bar{H}}$ | $K_{\bar{H}}$ | $\langle J_{H\gamma}, \bar{K}_{H\gamma} \rangle_F$ |
| $DOF_S$ | $p_w$ | $r_v$ | $J_S$ | $K_S$ | $\langle J_{S\gamma}, \bar{K}_{S\gamma} \rangle_F$ |

**Definition 11.** Holding Process [13]–[17]:A transportation independent process $p_{\varphi g} \in P_\varphi$ that holds artifacts during the transportation from one buffer to another.

**Example 2.** In production system, holding processes can be introduced to differentiate between holding different geometries. For example, two robots may have different end-effectors [13]–[17]. In water distribution systems, they can be introduced to differentiate between pipes that hold different types of water (e.g. potable and wastewater). In power grids, they can be used to differentiate transmission lines of different voltage level. In transportation systems, they have been used to differentiate between electrified and non-electrified roads [39].

In such cases, a knowledge base $J_\varphi$ and a constraint matrix $K_\varphi$ of size $\sigma(P_\varphi) \times \sigma(R)$ are constructed to capture the holding capabilities of the LFES. Then, the refined scleronomic transportation knowledge base $J_{\bar{H}}$ and constraints matrix $K_{\bar{H}}$ are formed using the kronecker tensor product and column selection [13]–[15], [15]–[17].

$$\begin{aligned} J_{\bar{H}} &= \left[ J_\varphi \otimes \mathbf{1}^{\sigma(P_\eta)} \right] \cdot \left[ \mathbf{1}^{\sigma(P_\varphi)} \otimes J_{\bar{H}} \right] \\ K_{\bar{H}} &= \left[ K_\varphi \otimes \mathbf{1}^{\sigma(P_\eta)} \right] \cdot \left[ \mathbf{1}^{\sigma(P_\varphi)} \otimes K_{\bar{H}} \right] \end{aligned} \tag{17}$$

where $\mathbf{1}^n$ is a ones vector of length n.

Intuitively, the sequence-independent structural degrees of freedom measure the number of ways that all of the system processes may be executed. They provide a flexible expression of LFES capabilities in the design and operational phases. From an axiomatic design perspective, the usage of knowledge bases facilitates further detailed engineering design [11], [26], [27]. The constraints matrix captures the potential for resource breakdowns and inflexibly implemented processes either physically or informatically in associated control and management structures. Additionally, from a graph theory perspective, the knowledge base forms a bipartite graph which may experience node or edge addition or elimination during operation. Finally, the mathematical form of the structural DOF measure matches the form of the mechanical DOF measure in Equation 4. The conceptual ties to mechanical degrees of freedom suggest that useful results from this field can potentially be applied to LFESs [12]–[20].

### C. Sequence-Dependent Structural Degrees of Freedom

The previous subsection recalled the development of sequence-independent structural degrees of freedom. A LFES, however, has constraints that introduce dependencies in the sequence of actions. A new measure is required for the sequence-dependent capabilities of the LFES [12]–[20].

**Example 3.** Reconsider Figure 3. Each solid line between activities represents a feasible sequence or pair of activities. Unconnected activities effectively have a design constraint that prohibits their sequential operation.

**Definition 12.** LFES Sequence-Dependent Structural Degrees of Freedom [12]–[20]: The set of independent pairs of actions that completely describe the system language.

In other words, the system language $\mathbb{L}$ can be described equally well in terms of the Kleene closure [38] of the

Table III: Types of Sequence-Dependent Production Degree of Freedom Measures [13], [14], [16], [17]

| Type | Measures | Processes | Resources | Knowledge Base | Constraint Matrix | Perpetual Constraint | Measure Function |
|---|---|---|---|---|---|---|---|
| **I** | $DOF_{MM\rho}$ | $P_\mu P_\mu$ | $M, M$ | $J_{MM\rho} = \left[J_M \cdot \bar{K}_M\right]^V \left[J_M \cdot \bar{K}_M\right]^{VT}$ | $K_{MM\rho}$ | $K_1 = K_2$ | $\langle J_{MM\rho}, \bar{K}_{MM\rho}\rangle_F$ |
| **II** | $DOF_{MH\rho}$ | $P_\mu P_\eta$ | $M, R$ | $J_{MH\rho} = \left[J_M \cdot \bar{K}_M\right]^V \left[J_H \cdot \bar{K}_H\right]^{VT}$ | $K_{MH\rho}$ | $k_1 - 1 = (u_1 - 1)/\sigma(B_S)$ | $\langle J_{MH\rho}, \bar{K}_{MH\rho}\rangle_F$ |
| **III** | $DOF_{HM\rho}$ | $P_\eta P_\mu$ | $R, M$ | $J_{HM\rho} = \left[J_H \cdot \bar{K}_H\right]^V \left[J_M \cdot \bar{K}_M\right]^{VT}$ | $K_{HM\rho}$ | $k_1 - 1 = (u_1 - 1)\&\sigma(B_S)$ | $\langle J_{HM\rho}, \bar{K}_{HM\rho}\rangle_F$ |
| **IV** | $DOF_{HH\rho}$ | $P_\eta P_\eta$ | $R, R$ | $J_{HH\rho} = \left[J_H \cdot \bar{K}_H\right]^V \left[J_H \cdot \bar{K}_H\right]^{VT}$ | $K_{HH\rho}$ | $(u_1 - 1)\%\sigma(B_S) = (u_2 - 1)/\sigma(B_S)$ | $\langle J_{HM\rho}, \bar{K}_{HM\rho}\rangle_F$ |
| **ALL** | $DOF_\rho$ | $PP$ | $R, R$ | $J_\rho = \left[J_S \cdot \bar{K}_S\right]^V \left[J_S \cdot \bar{K}_S\right]^{VT}$ | $K_\rho$ | All of the Above | $\langle J_{HM\rho}, \bar{K}_{HM\rho}\rangle_F$ |

sequence-independent and sequence-dependent structural degrees of freedom.

$$\mathbb{L} = E_S^* = Z^* \tag{18}$$

The calculation of sequence dependent structural degrees of freedom closely follows the approach in the previous section. The strings $z_{\psi_1\psi_2} = e_{w_1v_1}e_{w_2v_2} \in Z$ can be captured succinctly in a *rheonomic* (i.e sequence-dependent) LFES knowledge base.

**Definition 13.** LFES Rheonomic knowledge base [16]–[20]: A square binary matrix $J_\rho$ of size $\sigma(P)\sigma(R) \times \sigma(P)\sigma(R)$ whose element $J(\psi_1, \psi_2) \in \{0, 1\}$ is equal to one when string $z_{\psi_1,\psi_2}$ exists. It may be calculated directly as

$$J_\rho = \left[J_S \cdot \bar{K}_S\right]^V \left[J_S \cdot \bar{K}_S\right]^{VT} \tag{19}$$

Here, the sequence independent structural degrees of freedom are explicitly vectorized as has been done with mechanical degrees of freedom in Equation 5. Interestingly, and equally fundamental, $J_\rho$ is an adjacency matrix where the sequence independent structural degrees of freedom are treated as mutually connected nodes.

The development proceeds with the introduction of rheonomic constraints.

**Definition 14.** LFES Rheonomic Constraints Matrix $K_\rho$ [16]–[20]: a square binary constraints matrix of size $\sigma(P)\sigma(R) \times \sigma(P)\sigma(R)$ whose elements $K(\psi_1, \psi_2) \in \{0, 1\}$ are equal to one when string $z_{\psi 1 \psi 2}$ is eliminated.

Unlike its scleronomic counterpart where a zero matrix is possible, the rheonomic production constraints matrix has the perpetually binding constraints described in Table III. These ensure that the origin and destination of consecutive events match. Accurately keeping track of these constraints simultaneously is challenging. The final calculation of these minimal constraints is most easily implemented in a scalar fashion using FOR loops while adhering to the following relationships of indices. $\psi = \sigma(P)(v-1) + w$. $v = k$ $\forall k = [1 \ldots \sigma(M)]$. $w = [\sigma(P_\eta)(g-1) + u] + j$ [16], [17].

It follows that the number of LFES sequence-dependent degrees of freedom is [16], [17]:

$$DOF_\rho = \sigma(\mathcal{Z}) = \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} [J_\rho \ominus K_\rho](\psi_1, \psi_2) \tag{20}$$

$$= \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} [A_\rho](\psi_1, \psi_2) \tag{21}$$

As with sequence-indepedent structural degrees of freedom, sequence-dependent structural degrees of freedom may be classified in terms of their transformational and transportational variants. The calculation of the four types of measures is summarized in Table III and maintains an intuitive symmetry [16], [17]. In practice, the formation of the associated constraints matrices $K_{MM\rho}$, $K_{MH\rho}$, $K_{HM\rho}$,$K_{HH\rho}$ is an extra computational expense if $K_\rho$ has already been formed. Instead, the associated rheonomic production degree of freedom measures can be calculated by the appropriate replacement of $J_M$ or $J_{\bar{H}}$ with a zero matrix in Equation 15 [16], [17].

Intuitively, the sequence-dependent structural degrees of freedom measure the number of ways that pairs of system processes may be executed. Like their sequence-independent counterparts, they provide a flexible expression of the LFES capabilities in the design and operational phases. From an axiomatic design perspective, the rheonomic knowledge base and constraints matrices can be used to describe the potential reconfiguration of system processes and resources [13]–[17]. From a graph theory perspective, Equation 20 shows that the rheonomic knowledge base and constraints matrix form the adjacency matrix $A_\rho$ between nodes defined as structural degrees of freedom. The mechanical system analogy of $A_\rho$ was found in Equation 9 as part of the system's Lagrangian. As expected, $A_\rho$ is fundamental to the behavior of the LFES be it of discrete-event or continuous time nature.

### *D. Service-Specific Structural Degrees of Freedom*

While it is important to quantify the capabilities of a LFES, it is even more important to assess how well these capabilities are matched to the services that it intends to offer. This is especially important in the context of resilience measurement where either the required services or the existing capabilities may be changed. Furthermore, resilience is often measured with respect to a certain performance property [40]; thus implicitly defining one or more services of interest. This subsection builds upon the efforts of the previous sections to develop service-specific structural degrees of freedom. Intuitively speaking, the required services select out the sequence-independent structural degrees of freedom and the mathematical form of the associated measure is developed on that basis. To begin, the set of services is simplistically modeled so that the service-specific structural degree of freedom measures may be developed later.

*1) Service Modeling:* Before a treatment of service-specific structural degrees of freedom can be initiated, a systematic approach to describing services is required. For the sake of simplicity, the scope of this work restricts its scope to a class of services that do not require the mixing/assembly of heterogeneous artifacts, or the disjoining/separation of the same. The interested reader is referred to [14], [15], [17] for work that address such types of services.

A LFES may provide a set of services $L = \{l_1, \ldots, l_{\sigma(L)}\}$ where each service $l_i$ has its associated set of service activities $e_{xl_i} \in E_{l_i}$ which when all are completed result in the delivery of the service.

**Definition 15.** Service Activity [14]–[17]: A specific transformation process that may be applied as a part of larger service.

Consequently, the delivery of the service is described as a sequence of service activities [14]–[17]:

$$z_{l_i} = e_{x_1 l_i} e_{x_2 l_i} \ldots e_{x_{\sigma(E_{l_i})} l_i} \tag{22}$$

**Example 4.** Table IV provides concrete examples of services for production, transportation, power grid and water distribution systems.

Table IV: Example System Services in LFESs

| | |
|---|---|
| **Transportation**: | {Enter passenger at the origin station, Exit the passenger at the destination} |
| **Power Grid**: | {Generate electricity at the origin, Consume the electricity at the destination} |
| **Water Distribution**: | {Extract the water at the origin, Treat the water, Degrade the water through use, Dispose of the water at the destination} |
| **Production**: | {Enter the part to an input buffer, Mill the part, Drill a hole in the part, Polish the part, Exit the part from an output buffer} |

These examples offer a number of fundamental insights. First, it is important to note that this work implicitly assumes that the LFES is an *open* system. As such all of the services described above begin with an *entrance* of some artifact and end with its *exit*. Furthermore, water distribution and production systems require more careful thought given the potential heterogeneity of transformation processes and their artifacts. Finally, all of the systems do not fundamentally require transportation processes. All of the transformation processes required by the service could be conceivably realized in one location. This generalization is true even for transportation systems for the trivial case that the passenger's desired origin and destination are the same.

*2) Service Activity Feasibility:* The feasibility of a given service on an activity-by-activity basis follows straightforwardly from the introduction of two feasibility matrices.

**Definition 16.** Service Transformation Feasibility Matrix $\Lambda_{\mu i}$ [14]–[17]: For a given service $l_i$, a binary matrix of size $\sigma(E_{l_i}) \times \sigma(P_\mu)$ whose value $\Lambda_{\mu i}(x, j) = 1$ if $e_{xl_i}$ realizes transformation process $p_{\mu j}$.

**Definition 17.** Service Transportation Feasibility Matrix $\Lambda_{\gamma i}$ [14]–[17]: A binary row vector of size $1 \times \sigma(P_\gamma)$ whose value $\Lambda_{\gamma i}(g) = 1$ if product $l_i$ can be held by holding process $p_{\gamma g}$.

Note that because the service model only includes activities of a transformative nature, $\Lambda_{\gamma i}$ only marks feasibility between the service as a whole and the set of holding processes [14]–[17].

*3) Calculation of Service-Specific Structural Degrees of Freedom:* From these definitions it is straightforward to measure the number of LFES service-specific structural degrees of freedom. $\Lambda_{\mu i}$ and $\Lambda_{\gamma i}$ must first be used to produce a number of "selector" matrices that are equal in size to their corresponding LFES (scleronomic) knowledge base. Which one is used depends on the scope of interest; be it the type of process (e.g. transformation or transportation) or the desired scale (i.e. a single service activity, a whole service, or the whole set of services). Table V summarizes the definition and formulation of the multiple types of service selector matrices [16], [17].

Table V: Types of Service Selector Matrices [14]–[17]

| **Symbol** | **Formula** | **Scope** |
|---|---|---|
| $\Lambda_{Mxi}$ | $\left[e_x^T \Lambda_{\mu i}\right]^T \mathbf{1}^{\sigma(M)T}$ | Service Activity – Transformation |
| $\Lambda_{Mi}$ | $\left[\bigvee_{x}^{\sigma(E_L)} \Lambda_{\mu i}\right]^T \mathbf{1}^{\sigma(M)T}$ | Service – Transformation |
| $\Lambda_{ML}$ | $\left[\bigvee_{i}^{\sigma(L)} \bigvee_{x}^{\sigma(E_L)} \Lambda_{\mu i}\right]^T \mathbf{1}^{\sigma(M)T}$ | Service Line – Transformation |
| $\Lambda_{Hi}$ | $\left[\Lambda_{\gamma i} \otimes \mathbf{1}^{\sigma(P_\eta)T}\right]^T \mathbf{1}^{\sigma(R)T}$ | Service – Transportation |
| $\Lambda_{Hi}$ | $\left[\left[\bigvee_{i}^{\sigma(L)} \Lambda_{\gamma i}\right] \otimes \mathbf{1}^{\sigma(P_\eta)T}\right]^T \mathbf{1}^{\sigma(R)T}$ | Service – Transportation |
| $\Lambda_{Sxi}$ | $\left[\begin{array}{c\|c} \Lambda_{Mxi} & \mathbf{0} \\ \hline \multicolumn{2}{c}{\Lambda_{Hi}} \end{array}\right]$ | Service Activity – Transformation & Transportation |
| $\Lambda_{SMxi}$ | $\left[\begin{array}{c\|c} \Lambda_{Mxi} & \mathbf{0} \\ \hline \multicolumn{2}{c}{\mathbf{0}} \end{array}\right]$ | Service Activity – Transformation |
| $\Lambda_{Si}$ | $\left[\begin{array}{c\|c} \Lambda_{Mi} & \mathbf{0} \\ \hline \multicolumn{2}{c}{\Lambda_{Hi}} \end{array}\right]$ | Service – Transformation & Transportation |
| $\Lambda_{SHi}$ | $\left[\begin{array}{c\|c} \mathbf{0} & \mathbf{0} \\ \hline \multicolumn{2}{c}{\Lambda_{Hi}} \end{array}\right]$ | Service – Transportation |
| $\Lambda_{SL}$ | $\left[\begin{array}{c\|c} \Lambda_{ML} & \mathbf{0} \\ \hline \multicolumn{2}{c}{\Lambda_{HL}} \end{array}\right]$ | Service Line – Transformation & Transportation |

From these definitions, it is straightforward to measure the number of LFES service-specific structural degrees of freedom [16], [17].

$$DOF_{LS} = \langle \Lambda_{SL} \cdot J_S, \bar{K}_S \rangle_F \tag{23}$$

As mentioned at the beginning of the section, and entirely in agreement with the analogy in Section II-C3, this intuitive form of service-specific degrees of freedom shows that the services effectively select out the structural degrees of freedom provided by the LFES [16], [17].

### E. Conclusion: Relevance of Structural Degrees of Freedom to Resilience Measurement

Resilience measurement was introduced in Section I as an indirect measurement process that required an intermediate model. In this paper, the concept of structural degrees of freedom was developed based upon axiomatic design. The LFES knowledge bases and constraint matrices provide a quantitative description of "reconfiguration" potential; be it an arbitrary disruption or restoration of the system's capabilities. Such a reconfiguration processes would be mathematically described by [13], [16], [17], [41]:

$$(J_S, K_S, K_\rho) \rightarrow (J'_S, K'_S, K'_\rho) \tag{24}$$

In the sequel to this paper, this arbitrary transformation of the systems capabilities is used to give static measures of resilience.

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