

Static Resilience of Large Flexible Engineering Systems: Part II–Axiomatic Design Measures

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Abstract—Our modern life has grown to depend on many and nearly ubiquitous large complex engineering systems. In recent years, many disciplines have seemingly come to ask the same question: “In the face of assumed disruption, to what degree will these systems continue to perform and when will they be able to bounce back to normal operation?”. This paper seeks to partially fulfill this need with *static* resilience measures for *large flexible engineering systems* based upon an axiomatic design model. Given that the measurement of resilience is an indirect measurement process based upon models and formulaic measures, this two part paper is similarly organized. In Part I, the model is developed upon graph theory, axiomatic design for large flexible engineering systems (LFESs), and a tight analogy between mechanical systems and LFESs. Central to the development is the concept of structural degrees of freedom as the available combinations of systems processes and resources which individually describe system capabilities or sequentially give a sense of the skeleton of a system’s behavior. In Part II, the structural degree of freedom model is used to enumerate the service paths through a LFES along which valuable artifacts flow. The work then compares the value and quantity of service paths before and after a disruption as measures of static resilience – or survivability. A full illustrative example from the production system domain is provided. It is followed by a thorough discussion of the proposed resilience measures relative to the recent literature.

Index Terms—resilience, large complex systems, axiomatic design, graph theory, resilient systems, resilience measurement

I. INTRODUCTION

Our modern life has grown to depend on many and nearly ubiquitous large complex engineering systems [1]. Transportation, water distribution, electric power, natural gas, healthcare, manufacturing and food supply are but a few. These systems are characterized by an intricate web of interactions within themselves [2] but also between each other [3]. Our heavy reliance on these systems coupled with a growing recognition that disruptions and failures; be they natural or man-made; unintentional or malicious; are inevitable. Therefore, in recent years, many disciplines have seemingly come to ask the same question: “How *resilient* are these systems?” Said differently, in the face of assumed disruption, to what degree will these systems continue to perform and when will they be able to bounce back to normal operation [4]. Furthermore, the major disruptions of 9/11, the 2003 Northeastern Blackout, and Hurricane Katrina and Sandy has caused numerous agencies

to make resilient engineering systems a central policy goal [5]–[16].

Naturally, a large body of academic literature has developed on the subject across multiple disciplines. These include ecological [17], economic [18], organizational [19], [20], network [21], psychological [22] and socio-ecological [23] resilience [4]. Not surprisingly, a number of reviews [5], [22], [24], [25] on the topic have found that these contributions while complementary are not necessarily in agreement. The emerging field of resilience engineering, therefore, is still developing and requires formal quantitative definitions and frameworks [4], [5]. A key element to such rigorous approaches is the development of resilience measures which many, even recently, have identified as an area for concerted effort [4], [5], [22], [24], [26]–[29]. Such resilience measures would not only quantify resilience but could also inform designers and planners in advance how to best improve system resilience.

A. Contribution

This paper seeks to partially fulfill this need with *static* resilience measures for *large flexible engineering systems* based upon an axiomatic design model [30]. Much of the resilience measurement literature divides the life cycle property into two complementary aspects: a static “survival” property which measures the degree of performance after a disruption, and a dynamic “recovery” property which measures how quickly the performance returns to normal operation [1], [4], [5], [24], [26], [28], [29]. Additionally, many resilience measures in the literature depend on traditional graph theoretic applications. The choice of axiomatic design over (traditional) graph theory allows the paper’s scope to expand from *homo-functional* to *hetero-functional* systems. The paper’s contribution builds upon previous work in which axiomatic design was also applied to reconfigurable manufacturing and transportation systems [31]–[39]. One notable theme in the prior work was the enumeration of paths in these large flexible engineering systems which will be used here in the resilience measure development.

B. Scope

This paper restricts its scope to large flexible engineering systems.

Definition 1. Large Flexible Engineering System (LFES) [30]: an engineering system with many functional requirements (i.e. system processes) that not only evolve over time, but also can be fulfilled by one or more design parameters (i.e. system resources).

The paper also addresses static resilience as a measure of the degree a system can continue to perform after disruption. The dynamic “recovery” nature of resilience is left for future work.

C. Paper Outline

The measurement of resilience is naturally an indirect measurement process [40]. Therefore, this two-part paper is guided by Figure 1. In Part I, the concept of structural degrees of freedom [31]–[39] was developed using axiomatic design models. In this paper, this model is used to propose two resilience measures. Section II begins the development of



Fig. 1: A Generic Indirect Measurement Process [40]

the static resilience measures for large flexible engineering systems. The developments are then demonstrated on an illustrative example in Section III. Section IV then provides a thorough discussion of the utility of the developments. Finally, Section V concludes the work.

II. DEVELOPMENT OF STATIC RESILIENCE MEASURES

The background provided by the previous two sections allow for the development of static resilience measures for large flexible engineering systems. Many resilience measures in the existing literature are based upon some form of calculation of the shortest path length through a traditional graph [41]–[44]. Similarly, the static resilience measures proposed in this work depend on paths through a graph as well. In contrast, however, the measures derive from the *number* rather than the *length* of those paths. Furthermore, the paths are through a graph based upon nodes defined as structural degrees of freedom rather than simply locations as is often done in traditional graph theory. This section proceeds in two parts. Section II-A enumerates the number of paths for a service offered by an LFES. Section II-B then proposes the static resilience measures.

A. Path Enumeration in Large Flexible Engineering Systems

The development of static resilience measures depends on the enumeration of the number of feasible paths for each of the services provided by the LFES. The first step in this calculation is the translation of the service string in Equation 22(Part I) to an equivalent string composed of structural degrees of freedom. Because the service feasibility of transformation and transportation processes is fundamentally different, the

new equivalent string must also alternate between the two types of degrees of freedom [33]–[36].

$$z = e_{\mu_{j_1} m_{k_1}} [e_{\gamma_{\varphi_1} r_{v_1}}]^{*D} e_{\mu_{j_2} m_{k_2}} [e_{\gamma_{\varphi_2} r_{v_2}}]^{*D} \dots e_{\mu_{j_{\sigma(E_{l_i})}} m_{k_{\sigma(E_{l_i})}}} \quad (1)$$

where the exponent of an activity is used to denote the number of times it is repeated, and $*D$ denotes the repetition between zero and D times. D is taken as the diameter of the transportation network of the LFES as defined in Definition 4(Part I). For simplicity, Equation 1 can be rewritten explicitly in terms of its repeating pattern:

$$\left[e_{\mu_{j_x} m_{k_x}} [e_{\gamma_{\varphi_1} r_{v_1}}]^{*D} e_{\mu_{j_2} m_{k_2}} \right]^{\sigma(E_{l_i})} \quad (2)$$

Example 1. The string in Equation 2 is overly general for most LFESs. It is useful to specialize it for each of the four example LFESs discussed in this work:

Transportation: The passenger must enter at the origin station, take a number of transportation degrees of freedom and then leave at the destination. Therefore, Equation 1 can collapse to:

$$e_{\mu_{j_1} m_{k_1}} [e_{\gamma_{\varphi_1} r_{v_1}}]^{*D} e_{\mu_{j_2} m_{k_2}} \quad (3)$$

Power Grid: The electricity must be generated at a power plant, pass through a number of power lines and then be consumed at a destination. Therefore, power grids also follow the string in Equation 3.

Production: Production system’s naturally minimize transportation degrees of freedom because they are viewed as non-value adding. Good production system design practice allows at most one transportation process between any two transformation degrees of freedom. Therefore, Equation 2 can collapse to:

$$\left[e_{\mu_{j_x} m_{k_x}} [e_{\gamma_{\varphi_1} r_{v_1}}] e_{\mu_{j_2} m_{k_2}} \right]^{\sigma(E_{l_i})} \quad (4)$$

Note that this string still encompasses the potential for successive transformation events because the transportation events include “null processes” where no motion occurs at a system resource.

Water Distribution: Taken alone, the potable water distribution network resembles the transportation and power grid systems as in Equation 3. However, when the water distribution system is taken to include storm drains, wastewater, and recycled water, then the full string in Equation 2 is required.

The next step in path enumeration is to rewrite Equation 2 in terms of sequence-dependent structural degrees of freedom using Equation 18(Part I).

$$\left[z_{\psi_{M_x} \psi_H} [z_{HH}]^{*D-1} z_{\psi_{M_{x+1}} \psi_H} \right]^{\sigma(E_{l_i})-1} \quad (5)$$

where $z_{\psi_{M_x} \psi_H}$, z_{HH} , and $z_{\psi_{M_{x+1}} \psi_H}$ are Type II, III and IV sequence-dependent structural degrees of freedom respectively. This relatively compact form explicitly states in terms of the sequence-dependent structural degrees of freedom the paths through a LFES that would make a given service feasible.

The enumeration of these paths is straightforward using Theorem 1(Part I). To that effect, an adjacency matrix is written for each of three types of sequence dependent structural degrees of freedom in Equation 5. This requires the application of Equation 20(Part I) as specialized by Tables 2-4(Part I). For any given service i ,

$$A_{MHx_1\rho} = [\Lambda_{SMx_1} \cdot J_{SM} \cdot \bar{K}_{SM}]^V [\Lambda_{SH} \cdot J_{SH} \cdot \bar{K}_{SH}]^{VT} \ominus K_\rho \quad (6)$$

$$A_{HH\rho} = [\Lambda_{SH} \cdot J_{SH} \cdot \bar{K}_{SH}]^V [\Lambda_{SH} \cdot J_{SH} \cdot \bar{K}_{SH}]^{VT} \ominus K_\rho \quad (7)$$

$$A_{HMx_2\rho} = [\Lambda_{SH} \cdot J_{SH} \cdot \bar{K}_{SH}]^V [\Lambda_{SMx_2} \cdot J_{SM} \cdot \bar{K}_{SM}]^{VT} \ominus K_\rho \quad (8)$$

where x_1 and x_2 are the indices of sequential service activities. $x_2 = x_1 + 1$. These adjacency matrices are then multiplied together following Equation 5 to give the number of paths through a LFES for a given service i

$$DOF_{P_i} = \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} [A_{P_i}](\psi_1, \psi_2) \quad (9)$$

where

$$A_{P_i} = \sum_{d=1}^{\mathcal{D}} \left[(A_{MHx\rho}) (A_{HH\rho}^{d-1}) (A_{HMx+1\rho}) \right]^{\sigma(E_{i_d})-1} \quad (10)$$

Note that the summation over d is required here because there may exist transportation paths of length zero up to $\mathcal{D} - 1$ between successive transformations [37].

Example 2. As expected, this formula is overly general for most LFESs. It is useful to specialize it for each of the four example LFESs discussed in this work:

Transportation/Power/Potable Water Distribution

$$A_{P_i} = \sum_{d=1}^{\mathcal{D}} \left[(A_{MHx\rho}) (A_{HH\rho}^{d-1}) (A_{HMx+1\rho}) \right] \quad (11)$$

which confirms the result in [37].

Production:

$$A_{P_i} = \left[(A_{MHx\rho}) (A_{HMx+1\rho}) \right]^{\sigma(E_{i_d})-1} \quad (12)$$

which confirms the result in [35], [36].

Remark. For the application of resilience measurement, it is often important to distinguish between the number of transportation paths and the number of *simple* transportation paths (i.e. without loops). Intuitively speaking, transportation loops represent a type of unuseful redundancy. For this purpose, it may be necessary to eliminate the number of loops from the calculation of Equation 10. This can be done straightforwardly by eliminating the diagonal at each power of $A_{HH\rho}$. Instead $A_{HH\rho}^{D-1}$ can be replaced with $A_{HHS\rho}^{D-1}$ which is defined recursively:

$$A_{HHS\rho}^n = \left[A_{HHS\rho}^{n-1} - \text{diag}(A_{HHS\rho}^{n-1}) \right] [A_{HHS\rho}] \quad (13)$$

$$A_{HHS\rho} = A_{HH\rho} - \text{diag}(A_{HH\rho})$$

B. Static Resilience Measures

The developing consensus view on resilience measurement in LFES is best described by Figure 2. The LFES provides

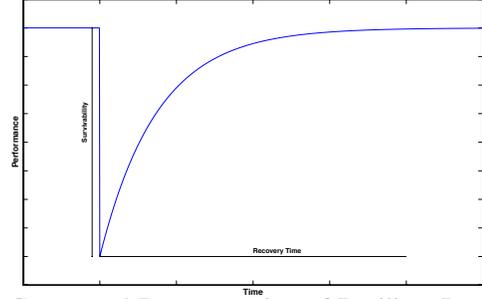


Fig. 2: Conceptual Representation of Resilient Performance

one or more services upon which a performance measure is based. A disruption of some kind then occurs which causes a performance degradation before the LFES is able to dynamically return to the initial service level. This work is concerned with *static* resilience measures which effectively quantifies *survivability* or the level of immediate degradation caused by the disruption. As stated at the conclusion of Part I, any given structural disruption can be represented as a change in (J_S, K_S, K_ρ) . Furthermore, the performance of the service depends on the existence of a *complete* path for its realization. Therefore, a static resilience measure can be defined as a function of the existence or number of paths that realize a service which are in turn a function of the structural degrees of freedom. The section proceeds by defining the concept of performance then proposing two resilience measures.

1) *Definition of Performance:* In the context of this work, the engineering performance of an LFES is addressed purely statically and structurally. Consider a service i , that begins with DOF e_{ψ_1} and ends with e_{ψ_2} . It delivers a quantity $Q_i(\psi_1, \psi_2)$ worth of a valuable artifact. The performance of that service is given by:

$$EP_i = \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} Q_i(\psi_1, \psi_2) \cdot bi[A_{P_i}(\psi_1, \psi_2)] \quad (14)$$

where $bi()$ is the binary function that returns 1 for all positive quantities and zero otherwise. Here, it is assumed that the number of paths for the service is not as important as the simple existence of such a path. As such, it assumes that each path is not capacity limited in Q . For the full engineering performance of the LFES over all services, it is necessary to linearly combine this measure with those of the other services.

$$EP = \sum_i^{\sigma(L)} \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} c_i Q_i(\psi_1, \psi_2) \cdot bi[A_{P_i}(\psi_1, \psi_2)] \quad (15)$$

where c_i is a measure of value of the i^{th} service such as utility, cost or profit that harmonizes the units of all Q_i .

In the context of this work, it is also useful to write the engineering performance measure explicitly in terms of the

knowledge base and constraint matrices (J_S, K_S, K_ρ) . With the change of notation $A_{P_i}(\psi_1, \psi_2) = A_{P_i\psi_1\psi_2}$,

$$EP = \sum_i^{\sigma(L)} \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} c_i Q_i(\psi_1, \psi_2) \cdot bi [A_{P_i\psi_1\psi_2}(J_S, K_S, K_\rho)] \quad (16)$$

Because the LFES's dynamics in terms of constitutive, continuity and compliance relations have not been modeled, Q_i is modeled as a constant rather than as a function of the structural degrees of freedom. Nevertheless, the ability to provide Q_i does require the existence of at least one service path and so studying the presence of such a path is a logical first step in resilience modeling and measurement.

2) *Actual Resilience*: The actual engineering resilience (AER) with respect to a particular disruption that takes the system through the transformation: $(J_{S_o}, K_{S_o}, K_{\rho_o}) \rightarrow (J_S, K_S, K_\rho)$ can now be defined straightforwardly.

$$AER = \frac{EP(J_S, K_S, K_\rho)}{EP(J_{S_o}, K_{S_o}, K_{\rho_o})} \quad (17)$$

This actual resilience measure benefits from the binary function $bi(\cdot)$. As expected, LFESs that exhibit some path redundancy for their services will not suffer from performance degradation. That said, the $bi(\cdot)$ also hides the effect of redundancy elimination caused by successive disruptions and so is not the most accurate predictor of the LFES' "health" towards future disruptions.

3) *Latent Engineering Resilience*: To address the limitations of the actual resilience measure, a latent engineering resilience measure is proposed. LER=

$$\frac{\sum_i^{\sigma(L)} \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} c_i Q_i(\psi_1, \psi_2) \cdot \frac{[A_{P_i\psi_1\psi_2}(J_S, K_S, K_\rho)]}{[A_{P_i\psi_1\psi_2}(J_{S_o}, K_{S_o}, K_{\rho_o})]}}{EP(J_{S_o}, K_{S_o}, K_{\rho_o})} \quad (18)$$

Here, the LER measure degrades gracefully with the transformation $(J_{S_o}, K_{S_o}, K_{\rho_o}) \rightarrow (J_S, K_S, K_\rho)$ because of the ratio of actual enumerated paths to the prior enumerated paths. It may be used to show how a given disruption eliminates some but not all of the paths of a given service.

III. ILLUSTRATIVE EXAMPLE

A production system is chosen for an illustrative example. As mentioned in Examples 4(PartI) and 1, production systems provide an extra level of functional heterogeneity that is less apparent in transportation, electric power and potable water distribution. To this end, the "Starling Manufacturing System" is taken as the test case for its functional heterogeneity and redundancy and its resource flexibility while maintaining a moderate size. The interested reader is referred to earlier references on structural degrees of freedom for further illustrative examples of this test case [31]–[36].

The system produces customized bird feeders from cylindrical wooden components. In this example, customers can choose between bird feeders of red and yellow color, and small and large radii. The wooden cylinders are turned for slots and

tabs, milled, laminated, and painted. All product variants have an injection molded dome roof and a base which doubles as a bird perch. These components are manually snapped onto the cylindrical bird feeders after production and are not further discussed in this example. Figure 3 shows a CAD model of one of the bird feeders.

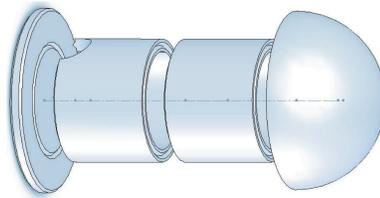


Fig. 3: CAD Model of Starling Bird Feeder

The production system itself is considered in three configurations that includes machining, lamination, and painting machines. Figure 4(a) shows the initial configuration, Figure 4(b) adds a second machining station, and Figure 4(c) makes all three value-adding resources redundant. Two shuttles transport the cylinders between the machines and buffer. The first has a flexible fixture which accommodates both radius sizes while the second can only carry cylinders of small radius.

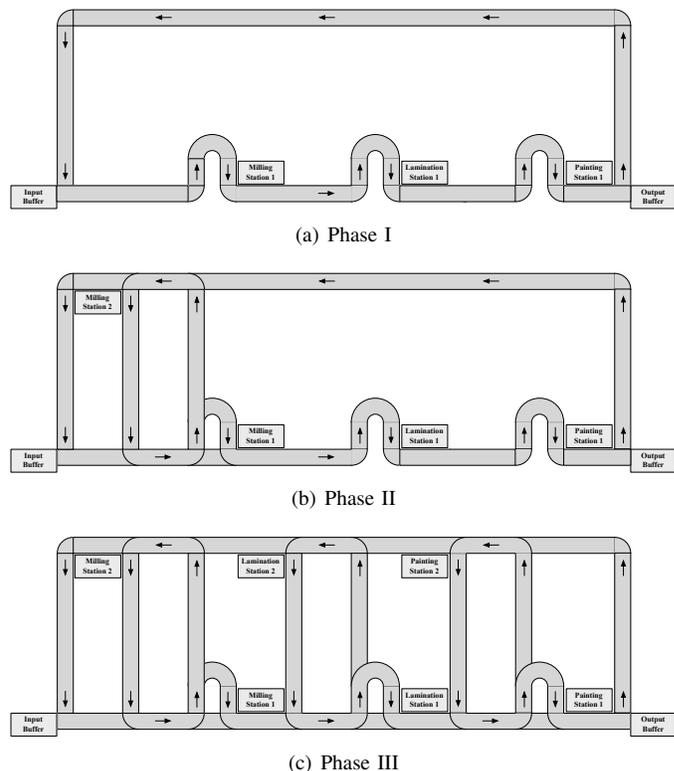


Fig. 4: Starling manufacturing system in three configurations

Next, the following sets of processes and resources are identified. In Phase I, $M=\{\text{Machining Station 1, Lamination Station 1, Painting Station 1}\}$. $B=\{\text{Input Buffer, Output Buffer}\}$. $H=\{\text{Shuttle A, Shuttle B}\}$. $P_M=\{\text{Lathe Tab, Lathe Slot, Mill Hole, Laminate, Paint Red, Paint Yellow}\}$.

$P_\eta = \{m_i m_j, m_i b_k, b_k m_i, b_k b_l\} \forall i, j = 1, 2, 3, k, l = 1, 2$. $P_\gamma = \{\text{Small Radial, Big Radial, Axial}\}$. The production processes and resources for the other system configurations may be determined analogously. Figure 5 presents the transformation, transportation, and holding knowledge bases for the Starling Manufacturing System in Phase I as monochrome images. It also shows the service transformation and transportation feasibility matrices for the small red bird feeder. For the sake of simplicity in the discussion, all four types of bird feeders are taken to sell the same quantities at the same price. The scleronomic constraints matrices are initially set to zero. The rheonomic constraints matrix has the minimal constraints identified in Section ???. The knowledge bases, constraints matrices, and service feasibility matrices for the other production system configurations and product variants can be readily formed by analogy. In order to measure the resilience of the production system, three types of disruptions are considered: a broken tab-lathing tool in Machining Station 1, a fault in Lamination Station 1, and a fault in Shuttle A which is capable of holding both small and large bird feeders.

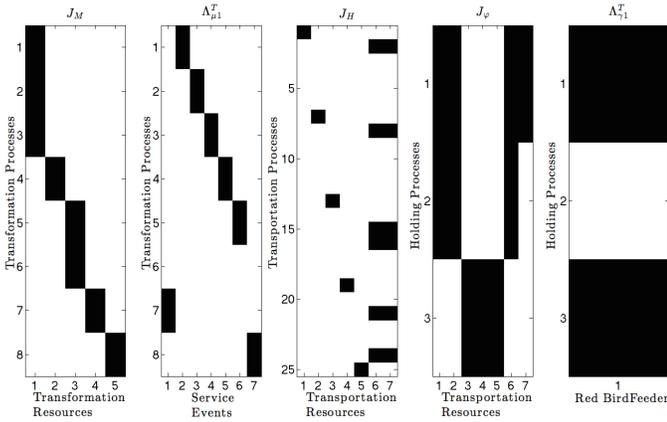


Fig. 5: Inputs for Phase I Starling Manufacturing System with a Small Red Bird Feeder: a.) Transformation Knowledge Base b.) Service Transformation Feasibility Matrix c.) Transportation Knowledge base d.) Holding Knowledge Base e.) Service Transportation Knowledge Base

The production paths for the three production system configurations under the various disruptions are shown in Table I. The corresponding values of actual and latent resilience are shown in Table II. Under normal operation, the number of production paths greatly expands as more redundancy is introduced into the system between Phases I and III. However, the utility of this redundancy is entirely different under different types of disruptions.

In Phase I, under normal operation, the small and large bird feeders exhibit a difference in the number of production paths because the latter has two available shuttles between transformation processes while the latter only has one. Thus, when Shuttle A fails, all of the large bird feeders can not be produced while the small bird feeders continue on Shuttle B. As expected, the 50% drop in AER reflects that 50% of the

Table I: Production Paths of Starling Product Line for Various Configurations & Faults

	Normal Stage 1	Broken Tool	Faulted Lamination	Faulted Shuttle A
Small Red	16	0	0	1
Small Yellow	16	0	0	1
Large Red	1	0	0	0
Large Yellow	1	0	0	0
	Normal Stage 2	Broken Tool	Faulted Lamination	Faulted Shuttle A
Small Red	96	80	0	4
Small Yellow	96	80	0	4
Large Red	4	3	0	0
Large Yellow	4	3	0	0
	Normal Stage 3	Broken Tool	Faulted Lamination	Faulted Shuttle A
Small Red	1152	576	576	32
Small Yellow	1152	576	576	32
Large Red	32	16	16	0
Large Yellow	32	16	16	0

Table II: Actual & Latent Resilience Values for Various Configurations & Faults

	Normal Stage 1	Broken Tool	Faulted Lamination	Faulted Shuttle A
AER	1	0	0	1/2
LER	1	0	0	1/32
	Normal Stage 2	Broken Tool	Faulted Lamination	Faulted Shuttle A
AER	1	1	0	1/2
LER	1	19/24	0	1/48
	Normal Stage 3	Broken Tool	Faulted Lamination	Faulted Shuttle A
AER	1	1	1	1/2
LER	1	1/2	1/2	2/189

birdfeeder types can no longer be produced. In contrast, the corresponding LER measure has fallen to 0.03125 (i.e. 1/32). This shows, that in reality, from a “system health” perspective, the disruption of Shuttle A eliminated production paths for all product variants; leaving only one for each type of bird feeder. Such a dedicated manufacturing line is highly brittle; and this is further shown by complete elimination of all production paths for the two other types of disruptions.

In Phase II, an additional milling station provides redundancy for three transformation processes: lathe tab, lathe slot, and mill hole. Setting aside the transportation system redundancy, the number of paths in normal operation should grow by $2^3 = 8$. However, it only grows by a factor of 4 for the large bird feeders because they can be transferred from the second milling station to the first but not vice versa. The disruption of Shuttle A has a similar effect in Phase II as in Phase I. The AER drops by 50% and the LER falls to an even smaller value to reflect that an even greater percentage of the production paths have been eliminated as a result of the disruption. That said, the addition of the redundant milling station has dramatically improved the AER and LER values in response to a broken tool. The zero AER and LER values in response to a fault in the lamination station highlight the production system’s vulnerability. Collectively, these results show that adding redundancy must be done judiciously. In this

case, tools may break often and so redundancy there may be extremely valuable. It's also worth noting that an additional milling station adds three new structural degrees of freedom whereas an additional lamination station would only add one. The proposed AER and LER measures thus have the potential to objectively inform investment decisions in how to upgrade the LFES.

In Phase III, all of the system's resources are redundant – but not for all products. The large bird feeders are still entirely disrupted by the failure of Shuttle A and this is similarly reflected in the AER and LER values. Because, the structural degree of freedom approach differentiates system resources in terms of their processes and their applicability to services, system vulnerabilities can be more easily resolved. Interestingly, the need for resilience in production systems *may* go counterflow to lean manufacturing trends where non-value adding processes were systematically eliminated. In this case, the lack of redundancy in the transportation system eliminates the utility of the other forms of redundancy for half of product line. For the other two disruptions, the results are predictable and intuitive. The AER measure provides a value of 1 to show that all the products can continue to be produced feasibly, while the LER measure gives a value of 0.5 to reflect the corresponding loss in the percentage of production paths.

IV. DISCUSSION

This section discusses some of the advantages of the proposed resilience measures in the context of the existing literature on the subject.

Implicit to many of the discussions in the resilience literature is the question: “How can this system be modified so that its performance doesn't degrade as much and returns to its original performance quickly?” The measurement of resilience as an output in response to a disruption as an input is a natural first step to answering such a question. However, when complex simulation packages are used for this purpose, they effectively become “black-boxes” that do not necessarily shed light as to *why* a particular disruption leads to a particular change in performance. Many recent works in the literature fit into this category [5], [24], [26], [28], [29], [45]. The predictive capability of such black-box measurement is only gained when a rigorous statistically-oriented system identification process has been completed. The degree of success of such an approach is increasingly uncertain as the system complexity grows. In contrast, a measurement method based upon (“white-box”) physical models can shed predictive insight instantly because the performance measure is functionally dependent on the measurables. In this regard, the proposed resilience measures are functionally dependent on the LFES's structural degrees of freedom as measurables. Like black-box approaches, they can measure the changes in the system performance. However, they have the added advantage of giving clear and intuitive advisory capability on *how* to best improve the system resilience.

The proposed resilience measures also present a useful level of abstraction in design and planning. Consider a designer who wishes a “resilient” mechanical system as the one described

in Section ?? . They may tune the values of masses and spring constants of the Lagrangian in Equation ?? but ultimately the resilience is primarily dependent on the *existence* of masses and springs and their connectedness. Focusing on the specifics of the equations of motion masks the underlying strengths and weaknesses of the underlying structure. Similarly, the engineering performance measure in Equation 15 can be detailed such that $Q_i(\psi_1, \psi_2)$ depends explicitly on the structural degrees of freedom. This requires the system-specific constitutive relations. The conventional industrial practice of N-1 contingency analysis conducted in power grids is exactly that [46]. While it remains absolutely necessary for power grid reliability studies, power grid operators will often say that the results of N-1 contingency analysis are much more dependent on the sparsity of the bus admittance matrix than the values of impedances and bus injections. Furthermore, in the early stages of design and planning, the specifics of the constitutive relations may not yet be known. The proposed measures allow for a resilience study as the system design or plan gains increasing levels of detail. Finally, because the proposed measures focus on the system structure which can be described rather abstractly, they also in a sense provide a greatest common factor approach to resilience measurement. More detailed resilience measurement approaches would likely have to deeply consider system-specific aspects.

While the proposed resilience measures incorporate many of the concepts found in the existing graph theoretic resilience measurement literature, the reliance on paths in structural degree of freedom networks presents four main advantages. First, some authors have discussed system resilience based upon measures of degree and eigenvector centrality [41], [44]. The illustrative example showed that the proposed resilience measures responded most substantially when more central nodes were eliminated. This work, however, in its focus on the value of service-specific paths distinguishes between nodes on the basis of their membership in paths of different value. Second, the literature has also proposed measures based upon connectedness; especially in transportation and communication networks [21], [27], [47]–[52]. The proposed resilience measures incorporate these results with the dependence on network paths; which fundamentally depend on graph connectedness. That said, many resilience improvement strategies *neither require nor encourage* graph connectedness. In power grids, for example, the recent literature encourages the design and operation of multiple microgrids which can connect and disconnect from each other while each serving their local demand for power [53], [54]. More fundamentally, electrical load can be served with onsite generation and without a power grid at all. Similarly, many regions do not require water grids because of the existence of local wells and lakes. Also similarly, production systems have long been designed and operated as manufacturing cells with either static or dynamic configurations. The concept of network connectedness in transportation and communication networks mostly arises because the demand for service often assumes that all possible origin-destination pairs are required. This is not necessarily

the case in other LFESs and the proposed resilience measures reflect these differences. Third, the literature has proposed resilience measures on the basis of shortest path lengths [41]–[44]. The proposed resilience measures recognize the need for paths because they essentially represent value chains from which performance can be measured. That said, the concept of shortest pathway is nearly irrelevant in LFESs where the valuable artifact travels exceptionally fast. For example, angular stability oscillations caused by an outage in Florida could be felt in Minnesota in less than three seconds [55]. Similarly, fiber optic networks rely on information transmission at the speed of light. Instead, the proposed resilience measures depend on the number of paths to quantify the potential for redundant ways of value delivery. Fourth, and finally, the networks presented in this work are based upon nodes as structural degrees of freedom rather than simply locations. This allows for the explicit description of multiple modes of transport and resources with multiple transformative functions. While an equally rich modeling framework can be achieved with the graph theoretic literature on node and edge colorings [56], to our knowledge this literature has never been applied to either the design of LFESs or to resilience measurement. In that regard, Axiomatic Design [30] and SysML [57] have a much more well-established history in both industrial practice and the academic literature.

V. CONCLUSION

This paper has proposed actual and latent engineering resilience measures for large flexible engineering systems. This development was founded upon graph theory, axiomatic design for large flexible engineering systems, and a tight analogy between mechanical systems and LFESs. Central to the development was the concept of structural degrees of freedom as the available combinations of systems processes and resources which could be measured individually to describe system capabilities or measured sequentially to give a sense of the skeleton of a system’s behavior. It was these sequence-dependent degrees of freedom that were used to enumerate service paths through a LFES. Each service path may be viewed as a value chain through an LFES along which quantities of valuable artifacts flow. Therefore, this work compared the value and quantity of service paths before and after a disruption as measures of static resilience – or survivability.

The reliance on graph theory *and* axiomatic design for large flexible systems present four main advantages for the resilience measures proposed in this work. The measures represent an indirect measurement processed based upon (“white-box”) physical models. As such, they not only measure the performance degradation of a particular disruption but they also offer a degree of advisory capability on how to best improve the system resilience. The measures also present a useful level of abstraction that may be applied in early design & planning decisions before the full details of the system behavior are known. The specific use of axiomatic design ensures that the relatively abstract graph theoretic descriptions

can be tied to well-established engineering design and systems engineering methodologies. Finally, from a graph theoretic point of view the use of axiomatic design knowledge bases overcomes many of the theoretical limitations to the graph theoretic resilience measures found in the existing literature.

The work presented in this paper leaves open many opportunities for future work. The first of which is a formal evaluation of the proposed measures on the basis of measurement theory. This has a practical scientific importance as it determines the types of statistics that can be applied to the resilience measures and further confines the scope of comparison. To our knowledge, and given the relatively early state of the literature, only one author has called for the need for such theoretical work [5]. More practically, this work can be extended to address the dynamic aspects of resilience. To that effect, the recent work on reconfiguration processes [58] may be applied as a promising avenue for future developments. Finally, the authors believe that this work has direct application to resilient control systems [59] in LFESs that possess a significant amount of control, automation and artificial intelligence.

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