A Hetero-functional Graph Analysis of Electric Power System Structural Resilience

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***Abstract*—Modern life has grown to be extremely dependent on electric power. As the world's services increasingly electrify, the resilience of the electric power grid is more important than ever. Current methods of studying electric power grid resilience generally fall in one of two categories: 1.) dynamic simulation methods and 2.) network science methods based upon graph connectedness. The latter use "lightweight" graph models while the former is considerably more computationally intensive. Though these methods provide valuable complementary insights, there is a need for analytical tools that balance analytical insight with computational complexity. This paper demonstrates, for the first time, a structural resilience analysis based upon the application of hetero-functional graph theory to electric power systems. These measures are of particular relevance to the grid's architectural transformation as it comes to accommodate distributed generation and meshed networks at the grid periphery. The paper concludes with a discussion of some of the key differences between existing resilience measures and those based upon a hetero-functional graph analysis.**

***Keywords*—Resilience, Hetero-functional graph theory, Electric power grid, Distributed generation**

I. Introduction

Modern life has grown to be extremely dependent on electric power. And yet, the shear size of the electric power grid means that disruptions are inevitable; be they severe weather, malicious attacks, or equipment failure[5]. As the world's services increasingly electrify, the need to "bounce back" from these disruptions in the form of resilience is more important than ever.

Current methods of studying electric power grid resilience generally fall in one of two categories: 1.) dynamic simulation methods and 2.) network science methods based upon graph connectedness. The first category relies on well-established power systems engineering techniques and their specific choice depends on the time-scale of the resilience analysis. For example, transient stability analysis is used to study disruptions in the 10-0.1Hz timescale[**?**]. Alternatively, grid operators conduct N-1 contingency analysis on the timescale of 5-60 minutes. In either case, these simulation-based techniques require a complete and appropriate dynamic model of the associated electric power system phenomena. Consequently, they 1.) are computationally intensive and 2.) only provide analytical insight after a full detailed-design iteration.

Alternatively, the network science methods use the connectedness in graph representations to make their resilience conclusions[9],[10]. Such highly abstract representations of system form are computationally light and provide immediate design and planning feedback. Basic graph models, however, lack an explicit representation system function and instead assume a homogeneity of function across the sets of nodes and edges. Consequently, such approaches (as this paper demonstrates) have limited utility in systems of heterogeneous function[3].

Though these two categories of methods provide valuable complementary insights, there is a need for analytical tools that balance analytical insight with computational complexity. In recent years, Hetero-Functional Graph Theory has emerged to quantitatively and explicitly represent the structure of systems with heterogeneous functionality. Rather than relying on an exclusively graph based description of system function, HFGT focuses on the links between "capabilities" that allocate system function to system form.

A. Contribution

This paper demonstrates, for the first time, a structural resilience analysis based upon the application of hetero-functional graph theory to electric power systems. These measures are of particular relevance to the grid's architectural transformation as it comes to accommodate distributed generation and meshed networks at the grid periphery. The paper concludes with a discussion of some of the key differences between existing resilience measures and those based upon a hetero-functional graph analysis.

B. Outline

The remainder of the paper is organized as follows. Section II provides a brief introduction to the background concepts used in this paper. Section III then introduces the IEEE power system test case used in the analysis. Section IV then presents the results of the hetero-functional graph analysis of electric power system resilience in contrast to more traditional network science methods. Section V brings the work to a conclusion.

II. Background and Methodology

This section provides a brief exposition of graph theory, hetero-functional graph theory, and relevant graph-based resilience measures as background concepts.

A. Graph Theory

In its most basic form, a graph is most commonly defined as a tuple $G = \{V, E\}$ where V is the set of (homogeneous) vertices and E is the set of (homogeneous) edges. Vertices often represent "point" facilities like power plants or substations. Meanwhile, edges often represent "connecting" facilities like power lines. Additionally, graphs are often classified as directed or undirected depending on whether their edges are specified as 1-way or 2-way[2]. Table I shows how graphs have been used in several common applications.

TABLE I
An example of Vertices and Edges in common networks

Graph	Vertices	Edges
Internet	Computer/Router	Wireless Data Connection
World Wide Web	Web Page	Hyperlinks
Power Grid	Plant or Substation	Transmission Line
Transport	Intersection	Roads
Neural Networks	Neurons	Synapse

The connectedness of a graph is often represented mathematically in an adjacency matrix A_{ij} of size $\sigma(V) \times \sigma(V)$ where the $\sigma()$ function provides the size of a set. The rows and columns of an adjacency matrix represent vertices and the elements within A_{ij} represent edges as follows:

$$A_{ij} = \begin{cases} 1 & \text{If an edge connects vertex } i \text{ to vertex } j \\ 0 & \text{Otherwise} \end{cases} \tag{1}$$

From a systems engineering perspective[**?**], it represents the system form aspect, or "what the system is." However, a complete representation of system architecture (or structure) must also define the system's function and their allocation to system form[**?**]. The distinction between system form and system structure is particularly important in hetero-functional systems.

B. Hetero-Functional Graph Theory

Hetero-functional graph theory was developed to provide an unambiguous and complete mathematical representation of system architecture. Consequently, it introduces a set of system processes to describe the system's functionality, or "what the system does". It also introduces a mapping of system function onto system form in what is called system concept. Each individual mapping of a process to a resource creates a unique capability, or structural degree of freedom, in the system concept. Linguistically, a capability appears as a "subject + verb + operand" sentence. For example, a power plant has the capability "Power plant A generates electricity". These capabilities (or structural degrees of freedom) become the vertices in a hetero-functional graph. Meanwhile, the edges in a hetero-functional graph become logical feasibility between one degree of freedom and another. Consequently, a reader can extract a story by following a hetero-functional graph across a series of edges. For example, " Power plant A generates electric power". "Transmission line B transports electric power from power plant A to substation C". "Substation C consumes electric power". To further contrast (traditional) graphs with hetero-functional graphs, Fig. 1 models the same hetero-functional system using both approaches[3].

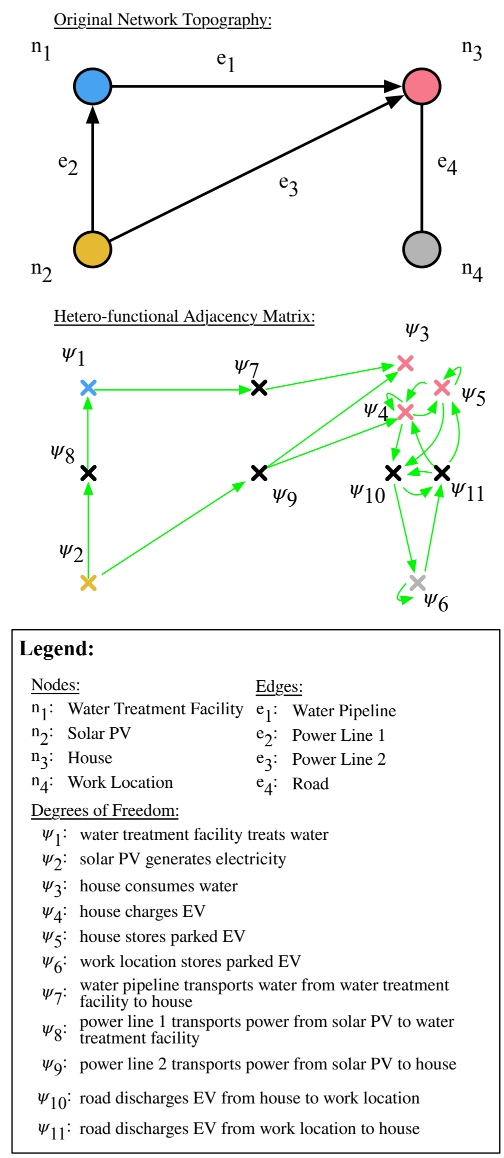

Fig. 1. A comparison of a traditional graph and a hetero-functional graph

C. Resilience Measures: Graph Theory

The network science study of resilience is grounded in centrality measures. Of these, degree centrality, closeness centrality, eigenvector centrality, and the large connected component are described here.

1) Degree Centrality: Degree centrality is the simplest centrality measure. It grants each vertex a score equal to the number of edges attached to it. When working with directed graphs, the degree centrality measure is further classified into in-degree and out-degree centrality; where the former counts the number of inward pointing edges and the latter counts the number of outward pointing edges. This paper restricts itself to the use of out-degree centrality to reflect the one-way outward flow of power in electric power distribution systems.

2) Closeness Centrality: The closeness centrality measure C(u) quantifies the average shortest path from one vertex to every other reachable vertex in the graph.

$$C(u) = \frac{n-1}{N-1} \frac{n-1}{\sum_{v=1}^{n-1} d(u,v)} \tag{2}$$

where $C(u)$ is the centrality of the node u, n is the number of reachable nodes, N is the number of nodes in the graph, and d is the length of the shortest path from u to node v. The term $\frac{n-1}{N-1}$ is applied to weight nodes in larger components with more importance that those in smaller components [2].

3) Eigenvector Centrality: Eigenvector centrality also determines a vertex's importance in relation to other vertices in the graph. Using eigenvalues and vectors, a node's centrality score is based upon not just its own degree centrality but also those of its neighbors. In matrix form the eigenvector centrality is calculated using (3).

$$Ax = \lambda x \tag{3}$$

where A is the adjacency matrix, λ is the largest eigenvalue of matrix A, and x is the eigenvector centrality for the set of nodes[2].

4) Largest Connected Component: Although not technically a centrality measure, the measurement of largest connected component serves to quantify the connectedness of a graph as it faces various forms of disruption. More specifically, it records the largest connected sub-graph (or component) within the original graph. Two vertices are defined to be in the same component if there is at least one path through the graph that connects them. Directed graphs distinguish between strongly and weakly connected components. The former requires the ability to loop back around to a starting node while weakly connected components simply require a single directed path between two nodes [2].

D. Resilience Measures: Hetero-functional Graph Theory

While the out-degree, closeness, eigenvector centrality, and largest component can all be applied to both a traditional graph and a hetero-functional graph they fail to take full advantage of the detail contained within the hetero-functional graph. Alternatively, a metric called the Actual Engineering Resilience (AER) was designed to exploit the detail of the hetero-functional graph. As a result it is only applicable to a hetero-function graph.

The paper [4] has described the derivation of the actual engineering resilience measures based on Axiomatic Design as it applies to hetero-functional Graph theory. This paper highlights the most important concepts but does not provide explicit directions to reproduce the AER formula.

The AER calculates how many services are deliverable. A service consists of a transformation process followed by any number of transportation processes then concluded by another transformation process. In this paper the service "Generate electric power"→"Transport electric power"→"Consume electric power" was tracked. By measuring the number of services delivered, a greater insight into the performance of the electric power grid is gained in comparison to basic node and edge distributions.

As presented in the paper [4], three adjacency matrices are required to calculate AER. One matrix is created for each type of structural degree of freedom sequence. The matrix A_{MH} tracks the sequence of transformation to transportation. The matrix A_{HH} tracks the sequence of transportation to transportation. The third matrix A_{HM} tracks the sequence of transportation to transformation. These matrices are multiplied together in (4) to determine the number of paths a service can be delivered across.

$$A_P = \sum_{d=1}^{D} [(A_{MH})(A_{HH}^{d-1})(A_{HM})] \tag{4}$$

By counting the number of non-zero elements in A_P the number of deliverable services is obtained. The summation over d is required as there exists D possible sequential transportation to transportation processes. As called for by the paper [4], The value D thus represents the longest simple path through the graph, however since this is an NP-hard problem the value of D was set to 15 for this paper.

III. Test Case

The test case for this paper was based off an IEEE Power and Energy Society 123-Bus Feeder test case for solving unbalanced three phase radial systems [8]. This data set was chosen for its structure and scale. As the 123-Bus Feeder test case started with 123 vertices it had a scale large enough to see emerging patterns through disruptions but was small enough to visually confirm results. Additionally, the test case had a radial distribution structure that resembled suburban distribution networks [7]. The 123-Bus feeder test case was thus adopted and adapted for this study. The base case was adapted so it included 129 nodes and 122 edges. One node was designated as the interconnect to a transmission and thus had the functionality "generate electric power" while the remainder of the nodes had the functionality "consume electric power". To allow for flow reversals across lines in the presence of disruptions the distribution lines were designated an undirected.

To test the capabilities of the resilience metrics several test cases were developed off of the base case. These test cases included a meshed distribution, distributed generation, and meshed distribution with distributed generation models. The models with a meshed distribution had 32 additional distribution lines added to the structure. The models with distributed generation had "generate electric power" added to the functionality of 41 nodes. A depiction of the final test case with distributed generation and a meshed distribution system is shown in fig. 2. Each case was modeled as both a traditional graph and as a hetero-functional graph.

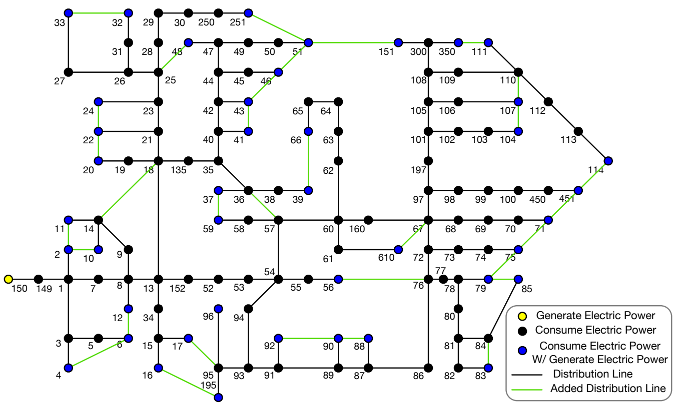

Fig. 2. The final test case with blue nodes designating distributed generation additions and green lines representing distribution line additions.

IV. Results

Nodal disruptions were applied to both the traditional and hetero-functional graph. The disruptions were applied randomly or through targeted disruptions based on centrality. To ensure the physics remained the same between the two graph theories when a node was removed from the traditional graph all corresponding nodes in the hetero-functional graph who's form was associated with the physical resource were also removed.

When measuring the out degree centrality of both graphs on the base test case under random disruptions, they presented similar characteristics. The average out-degree of each graph linearly decrease as nodes were removed. The random disruptions are depicted in Fig. 3.

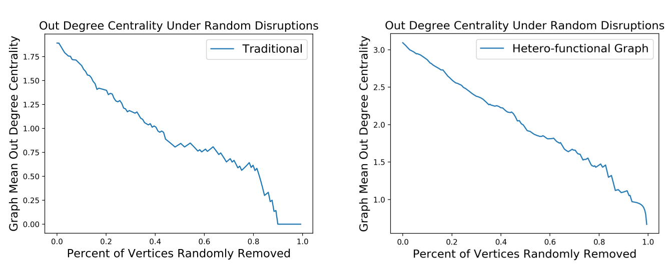

Fig. 3. The out-degree centrality of the base case under random disruptions for both the traditional and hetero-functional graphs.

Additionally, when monitoring the size of the largest connected component under targeted disruptions both the traditional graph and hetero-functional graph presented similar characteristics. The disruptions were targeted at nodes using the centrality of using out-degree, closeness, and eigenvector centrality. Since the nodes with the highest centrality were removed first, then reevaluated with each iteration, the most important and central nodes were removed first. As a result the largest component sees a large initial drop in size as major connections are removed. Similar to the random disruptions, Fig. 4 show that targeted standard network science centrality metric continue to hold similar results when applied to both the traditional graph and the hetero-functional graph.

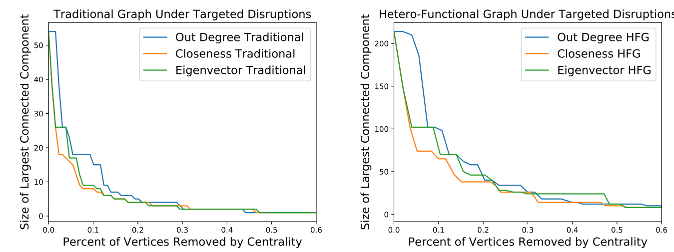

Fig. 4. The size of the largest component of the base case under targeted disruptions for both the traditional and hetero-functional graphs.

Under random disruptions the centrality of a traditional graph gradually decreases. While this shows the electric power grid's ability to function decreases it does not provide an accurate depiction of how damaged the electric power grid actually is. However, the AER applied to the hetero-functional graph under random disruptions demonstrates a much more drastic decrease in the amount of deliverable services. Fig. 5 shows how the AER is able to highlight the severe vulnerability of the base case's radial topology.

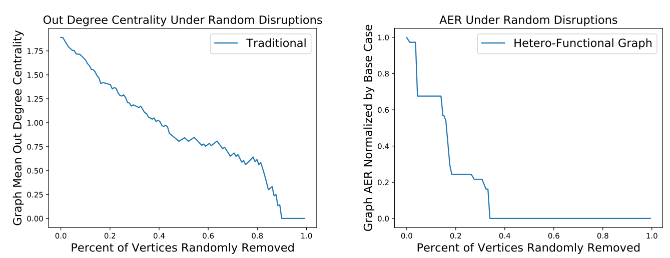

Fig. 5. The average out-degree centrality of the traditional graph base case under random disruptions and the AER of the hetero-functional graph base case under random disruptions.

As additional distribution lines are added onto the base case both the traditional graph's largest component and the hetero-functional graph's AER increase. By converting the radial electrical power grid into a meshed distribution system, both tradition connectivity and the AER report an increase in resilience of the electric power grid in fig. 6. Both methods are able to identify a change in the network and increased resilience because the additional lines are a purely formal change and can be incorporated into the traditional graph.

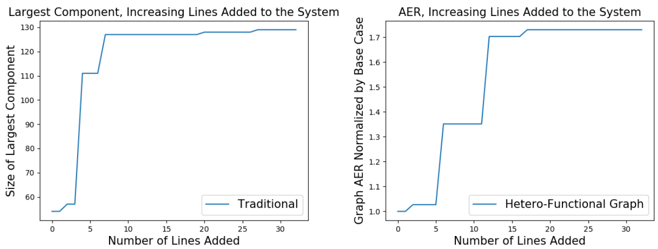

Fig. 6. Size of the largest component of a traditional graph and the AER of a hetero-functional graph as distribution lines are added to the electric power grid.

The addition of distributed generation to the existing nodes of the base case becomes a purely functional change to the electric power grid. Thus, traditional graphs are unable to capture the modifications made by the additional functionality as they only store form. The hetero-functional graph however, is able to incorporate these changes and reflect them in the AER. As a result the size of the larges component of the traditional graph remains constant while the AER of the hetero-functional graph increases with the addition of distributed generation. The deficiency of standard resilience measures and benefit of the AER is displayed in fig. 7.

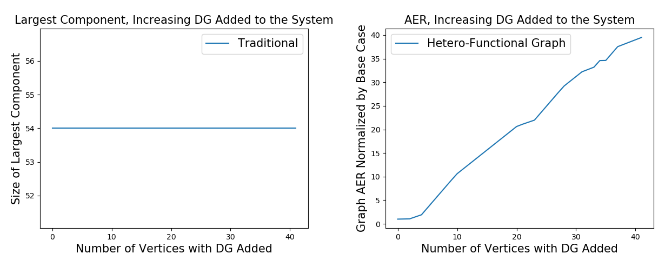

Fig. 7. Size of the largest component of a traditional graph and the AER of a hetero-functional graph as distributed generation functionality is added to the electric power grid.

V. Conclusion

This paper has shown that traditional graph theory and standard resilience metrics are incapable of modeling and measuring a complete set of advancements to an electric power grid such as distributed generation. These shortcomings are a direct result of traditional graphs being confined to the form of the network without the ability to directly describe functionality. Hetero-functional graph theory fills the need for a model that is able to handle function and form with it's fundamental allocation of function onto form. As the standard centrality metrics still provide valuable information on the system, it is worth noting that the hetero-functional graph continues to present the same characteristics as a traditional graph under the same measures. However, with the increased detail stored in hetero-functional graph theory, the platform for new resilience metrics such as the AER is formed. The AER follows and enumerates the number of services a network can deliver. By directly counting the number of deliverable services a more precise understanding of an electric power grid's resilience can be determined than with the more abstract centrality measures. Additionally, the AER has proven to be a more capable resilience metric especially when adding distributed generation. While the standard measures failed to show a change in resilience with added distributed generation, the AER showed a clear increase in resilience. As societies dependency on electric power increases it will become imperative that hetero-functional graph theory and the AER be used to plan a more efficient and resilient electric power grid.

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