An Axiomatic Design Approach to Passenger Itinerary Enumeration in Reconfigurable Transportation Systems

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Abstract—**Transportation systems represent a critical infrastructure upon which nations' economies and national security depend. As infrastructure systems, they must be planned and operated to accommodate the uncertain and continually evolving needs of their passengers and freight. New roads are planned or existing ones are closed for maintenance or due to operational breakdowns. Reconfigurable transportations systems are those which adapt to these changes quickly and efficiently. They are not over-designed with capabilities that may be left unused, instead capabilities are added only when needed; thus supporting the need for *resilient* infrastructure. An axiomatic design for large flexible systems approach is chosen as a methodology for its deep roots in engineering design. It addresses systems where the functionality not only evolves over time, but also can be fulfilled by one or more system resources, and is used here to enumerate passenger itineraries. This paper builds upon a recent work in which axiomatic design was used to develop a theory of degrees of freedom in transportation systems for their reconfigurable design and operation. The methodological developments are then demonstrated on a small subsection of the Mexico City transportation system to demonstrate its wide ranging utility in reconfigurability decision-making at the planning and operations time scales. Also further comparisons of axiomatic design to traditional graph theory are made indicating the mathematical basis of the former in the latter.**

Index Terms—**axiomatic design, transportation paths, transportation itineraries, Mexico City transportation system, reconfigurability, resilience, reconfigurable transportation systems, resilient transportation systems.**

I. Introduction

Transportation systems represent a critical infrastructure upon which nations' economies and national security depend. In the 1990s, transportation systems, became increasingly strained by the continually evolving needs of a growing population that has trended towards concentrating in cities for the past 100 years [1]. One particularly pertinent problem is the need to quickly find ways to reallocate and adjust the capacity and capabilities of transportation resources to the variants that need them most. Another key challenge is the transportation system's resilience in the face of unplanned disturbances, events, or disasters. Reconfigurability and resilience drivers

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can be found to varying degrees in many of the modes of transport: air, ship, rail, and road. Recently, decentralized reconfiguration strategies for reconfigurable transportation systems have emerged [2]. In order to achieve and support these solutions, it becomes necessary to model the evolution of the system architecture. The realization of these incremental changes requires decisions to be made in the operations and planning of transportation systems. This requirement causes a multi-dimensional engineering management problem which stakeholders have to find ways to address. To fulfill these needs, reconfigurable transportation systems are proposed as a possible solution. They are defined as:

Definition 1. Reconfigurable Transportation System [3], [4]: A system designed at the outset for rapid change in structure, in order to quickly adjust capacity and functionality in response to sudden changes in stakeholder requirements. Reconfigurable transportation systems are those in which new capabilities are added only when needed, and the system is not over-designed with capabilities that may be left unused.

This paper uses an axiomatic design approach called transportation degrees of freedom to enumerate the number of passenger itineraries in reconfigurable transportation systems [3], [4]; transportation systems with variable system architecture. The enumeration of passenger itineraries, and more generically paths through a network, has long been associated with network reliability and resilience [5]–[7]. Here, the axiomatic design based approach serves a number of additional purposes. First, because the enumerated passenger itineraries are set in terms of the evolving system architecture variables in both function and form, transportation system reconfigurations can be explicitly modeled. Second, the axiomatic design approach bridges the traditionally graph theoretic approaches to the engineering design community. These advantages are revisited more extensively in Section VI.

This paper serves to describe an axiomatic design approach to passenger itinerary enumeration and stress its novel importance to intelligent and reconfigurable transportation systems. The argument of the paper proceeds as follows. Section II provides the background to the methodological developments with brief introductions to graph theory [8]–[10] and its applications in intelligent transportations systems, axiomatic design for large flexible systems [11], and an analogy between transportation and mechanical systems [3], [4], [12]–[14]. Section III then recounts how axiomatic design has been used to develop the concept of transportation degrees of freedom as applied

to transportation and production systems [3], [4], [12]–[14]. Next, Section IV enumerates passenger itineraries as a measure called passenger degrees of freedom upon this foundation [3], [4]. Section V illustrates the methodological developments on a small subsection of the Mexico City transportation system [3], [4]. Section VI provides a novel discussion of the relation between axiomatic design and graph theory. Section VII then describes the reconfigurability applications of these measures in the planning and operations time scales [4], [12]. Finally, Section VIII concludes the work and proposes avenues for future work.

II. BACKGROUND

This section summarizes the methodological developments found in the existing literature in order to provide a foundation for the enumeration of passenger itineraries in the next section. The discussion proceeds in three steps. Section II-A gives a brief introduction to graph theory while Section II-B introduces the application of axiomatic design for large flexible systems to transportation systems. Section II-C then discusses a taxonomy of transportation system degrees of freedom as presented in earlier work.

A. Graph Theory in Transportation Networks

Graph theory is a long established field of mathematics with applications in many fields of science and engineering where artifacts are transported between physical locations [8]–[10]. A number of definitions are introduced for later use in the discussion.

Definition 2. A graph [9]: $G = \{V, E\}$ consists of a collection of nodes V and collection of edges E. Each edge $e \in E$ is said to join two nodes, which are called its end points. If e joins $v_1, v_2 \in V$, we write $e = \langle v_1, v_2 \rangle$. Nodes v_1 and v_2 in this case are said to be adjacent. Edge e is said to be incident with nodes v_1 and v_2, respectively.

Definition 3. A bipartite graph [9]: Graph G is bipartite if $V(G)$ can be partitioned into two disjoint subsets V_1 and V_2 such that each edge $e \in E(G)$ has one end point in V_1 and the other in V_2: $E(G) \subseteq \{e = \langle v_1, v_2 \rangle | u_1 \in V_1, \text{ and } u_2 \in V_2\}$.

Definition 4. A directed graph (digraph) [9]: D, consists of a collection nodes V, and a collection of arcs A, for which we write $D = (V, A)$. Each arc $a = \langle v_1, v_2 \rangle$, is said to join node $v_1 \in V$ to another (not necessarily distinct) node v_2. Vertex v_1 is called the tail of a, whereas v_2 is its head.

Definition 5. Adjacency matrix [9]: A, is binary and of size $\sigma(V) \times \sigma(V)$ and its elements are given by:

$$A(i,j) = \begin{cases} 1 & \text{if } \langle v_i, v_j \rangle \text{ exists} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where the operator $\sigma()$ gives the size of a set. Interestingly, $A^N(i,j)$ represents the number of traveler itineraries of N-steps between origin and destination [8].

Definition 6. Incidence matrix [9]: M of size $\sigma(V) \times \sigma(A)$ is given by:

$$M(i,j) = \begin{cases} -1 & \text{if } v_i \text{ is the head of arc } a_j \\ 1 & \text{if } v_i \text{ is the tail of arc } a_j \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Graph theory has found extensive usage in transportation networks. Most recently, the above definitions have been extended to multi-modal transportation systems in the form of the itinerary/journey planning problem, the Vehicle Routing Problem (VRP) and its variants, Demand Responsive Transport(DRT) or its broader version of Flexible Transport Services (FTS) [15]. The journey planning problem involves determining a path between an origin and destination. Common criteria for evaluating these journeys maybe total duration, number of transfers and cost [16]. Dynamic journeying problems using graph theory is discussed in [16]. The path may traverse through a sequence of stops on a given route and specified schedule. Such a route is then called an itinerary. The itinerary planning problem finds the itinerary which optimizes a set of criteria like the travel time, number of transfers and the total time, subject to scheduling constraints. Itinerary planning problems using graph theory are discussed in [17]. There is uncertainty related to the arrival and departure times at different stations and hence a journey planning problem is assisted with uncertainty analysis [16], which makes them easily applicable to FTS. DRT services are mainly related to door-to-door Dial-a-Ride services provided by statutory authorities [18]. Dynamic VRP introduced by Wilson and Colvin [19] involves a journey taken up dynamically, on request by a customer. Dynamic routing is more complex than its static counterpart as it requires real time knowledge of vehicle position and the ability to communicate with driver [20]. Further promising work on node and edge colorings by [21]–[24] are applicable to multimodal transport. The VRP, DTS, and DRT problems certainly require reconfigurability potential in the usage of transportation networks but not necessarily of the networks themselves [3]. This work focuses on the engineering design of the transportation network itself as a multi-modal system with reconfiguration potential. Examples of these applications are further discussed in Section VII.

While graph theory for decades has presented a useful abstraction of transportation networks for operations research, it has limitations from an engineering design perspective. One author writes:

> Interestingly, the fraction of *bona fide* engineers pursuing this approach has remained relatively small; it is mostly mathematicians, physicists and biologists who pursue this particular view of complex systems. This maybe because of the emphasis on analyzing systems "as they are" rather than on building systems that do not yet exist. It may also be that engineers have to focus on technical details and many of them remain suspicious of highly abstracted mathematical representations of systems such as system graph representations, where all nodes are essentially treated as equal [1].

The above definitions focus on the abstract form of a homogenous transportation network and less so the transportation functions itself. Furthermore, how the functions are realized is not explicitly stated. Unless generalized, such graph theoretic approaches are likely to have limitations in systems of heterogeneous function and form. Furthermore, because the system

functionality and its realizing form has been abstracted away, such approaches may not straightforwardly lend themselves to detailed engineering design of the system's component functions. This disconnect may impede intelligent control solutions that implement reconfigurable transportation system architectures [3].

B. Axiomatic Design for Large Flexible Systems

In contrast, axiomatic design of large flexible systems provides a natural engineering design description. Broadly speaking, axiomatic design originates from the field of machine tool design [11] but has been successfully applied to control systems [25], electronics [26], software [27], [28], intelligent manufacturing systems [12], [29], supply chains [30], organizational design [11], and human resources management [31], [32]. It has been specifically applied to transportation applications in the design of intersections [33]–[36], airport terminals [37], and shipping companies and ports [38]–[40]. This work adds to these works in transportation by expanding the scope to include the entire transportation system network.

To this end, the axiomatic design of large flexible systems proves a useful design tool. Suh defines large flexible systems as systems with many functional requirements that not only evolve over time, but also can be fulfilled by one or more design parameters [11]. In transportation systems, the set of functional requirements is taken as the set of transportation processes, $\mathbf{FR} = \{$Transportation Processes$\}$.

Definition 7. Transportation Process [12]: Given an origin station b_{y_1} and a destination b_{y_2} within a set of stations B, a transportation-resource-independent process $p_u \in P$ transports individuals *directly without stop* between b_{y_1} and b_{y_2}. A convention is adopted between the indices of stations and transportation processes such that:

$$u = \sigma(B)(y_1 - 1) + y_2 \tag{3}$$

There are $\sigma^2(B)$ such transportation processes of which $\sigma(B)$ are "null processes" where no motion occurs.

The set of design parameters is taken as the set of transportation resources $\mathbf{DP} = \{$Transportation Resources$\}$.

Definition 8. Transportation Resource [12]: A vehicle $r \in R$ capable of realizing one or more non-null transportation processes such as a bus or train.

C. Transportation Degrees of Freedom: An Analogy

The concept of degrees of freedom as applied to large flexible systems originated with research in automated reconfigurable manufacturing systems in which an analogy between mechanical and production degrees of freedom was drawn [12], [13]. Since then, the analogy has been specialized to transportation systems [3], [4]. For the sake of intuition development, the analogy is redrawn here.

Production system degrees of freedom arose from an analogy between mechanical and production systems that holds equally well for transportation systems [12], [13]. At the most basic level, a mechanical system is defined by its kinematics which is described by links and coordinates [41]. Links make

up the physical composition of a mechanical system. Similarly, transportation systems are composed of transportation resources. Coordinates are used to express the time-evolution of a continuous state which results in motion. Transportation systems have been similarly modeled in order to describe their system behavior [42], [43]. However, an event-driven evolution of discrete states is more appropriate for reconfigurable transportation system architecture. Cassandras and LaFortune [44] have previously drawn this analogy between coordinates for time-driven systems and events for event-driven systems. Finally, when analyzing multi-body mechanical systems, the number of coordinates is calculated based upon the number of combinations of dimensions and links less any applicable constraints [41]. For example, a fully free three-link system has 18 degrees of freedom: 6 dimensions for each of the three links. The analogy suggests that transportation system degrees of freedom would come from the feasible combinations of transportation processes and their associated resources less applicable constraints. Finally, mechanical degrees of freedom are classified as either scleronomic, i.e. time-independent, or rheonomic, i.e. time-dependent [41]. This suggests that event-driven systems' degrees of freedom would be scleronomic or rheonomic in relation to their sequence dependence.

III. TRANSPORTATION DEGREES OF FREEDOM

This section recounts how transportation degrees of freedom has been applied to transportation and production systems [3], [4], [12]–[14]. First, a measure of scleronomic transportation degrees of freedom is developed as a measure of the sequence-independent capabilities of a transportation system. Next, a measure of rheonomic transportation degrees of freedom is developed to address sequence-dependent capabilities.

A. Scleronomic Transportation Degrees of Freedom

The heart of scleronomic transportation degrees of freedom arises from the axiomatic design knowledge base for large flexible systems [12]. Its development is recounted here for clarity. Suh uses the large flexible system design equation notation:

$$\begin{aligned} FR_1 &\$(DP_1, DP_2, DP_3) \\ FR_2 &\$(DP_2, DP_3) \\ FR_3 &\$(DP_3) \end{aligned} \tag{4}$$

to signify that FR_1 can be realized by (indicated by the $\$$ symbol) design parameters DP_1, DP_2, or DP_3 [11]. Previous work reinterprets the design equation in Equation 4 in terms of a matrix equation using a boolean knowledge base matrix J which contains the systems degrees of freedom [12].

$$FR = J \odot DP \tag{5}$$

where matrix boolean multiplication $C = A \odot B$ is equivalent to $C(i,k) = \bigvee_j (A(i,j) \wedge B(j,k))$ and $\bigvee_j^n a_j = a_1 \vee a_2 \ldots a_{n-1} \vee a_n$ is the array-OR operation [45].

The transportation system knowledge base found in Equation 5 describes the transportation system's sequence-independent capabilities and is defined formally as follows.

Definition 9. Transportation System Scleronomic Knowledge Base [3]: Given a set of transportation processes P and a set

of transportation resources R, an event $\epsilon_{uv} \in \mathcal{E}$ (in the discrete event system sense) [44] can be defined for each feasible combination of production process p_u being realized by resource r_v. The Transportation System Scleronomic Knowledge Base J_S is a binary matrix of size $\sigma(P) \times \sigma(R)$ where element $J_S(u,v) \in \{0,1\}$ is equal to one when event ϵ_{uv} exists.

Interestingly, the axiomatic design knowledge base itself forms a bipartite graph [9] between the set of processes (e.g. functional requirements) and resources (e.g. design parameters).

Although these events/capabilities may exist in the transportation system, there may exist phenomena and/or artifacts to prevent them from being enabled in operation [3], [12]. These may include temporary vehicle breakdowns, line closures, or road detours. Additionally, the supporting automation and IT systems may prevent a given vehicle from fully realizing all of the routes for which it is physically capable. To model these phenomena, a number of discrete constraints can apply in the operational time frame so as to eliminate events from the event set. These constraints are said to be *scleronomic* as they are independent of event sequence. The existence of these discrete constraints can be described in a single binary matrix as defined.

Definition 10. Transportation System Scleronomic Constraints Matrix [3]: K_S of size $\sigma(P) \times \sigma(R)$ whose elements $K_S(u,v) \in \{0,1\}$ are equal to one when a constraint eliminates event e_{uv} from the event set.

In practice, these constraints can be be captured manually by examining if there exists a reason which prevents a given resource from realizing its associated transportation processes [12]. Alternatively, these constraints can be captured in automatic transportation systems from the existence of system faults or alert messages [12].

Within this work, and so as to not exaggerate the transportation system capabilities with null processes of remaining at the same station, K_S is used to eliminate these events by convention. This is done by adding a constraint wherever the index of a null process p_u satisfies Equation 3 and that $y_1 = y_2$. Equation 3 can be inverted to give the indices y_1 and y_2 from the transportation process index u [12].

$$y_1 = \mathrm{mod}((u-1)/\sigma(B)) \qquad (6)$$
$$y_2 = (u-1)/\sigma(B)$$

where $\mathrm{mod}(x,y)$ is the modulus (i.e. fractional part) of x with respect to y and $/$ is integer division. It follows that [3]

$$K_S(u,v) = \begin{cases} 1 & \text{if } \mathrm{mod}((u-1),\sigma(B)) = \\ & \qquad (u-1)/\sigma(B) \\ 0 & \text{otherwise} \end{cases} \qquad (7)$$

Or equivalently [3],

$$K_S = (\mathbf{I}^{\sigma(B)})^V \otimes \mathbb{1}^{\sigma(R)T} \qquad (8)$$

where $\mathbf{I}^m$ is the identity matrix of size $m \times m$, the A^V operation is shorthand for vectorization $vec()$ commonly implemented in MATLAB with the (:) operator, $\otimes$ is the kronecker tensor product, and $\mathbb{1}^m$ is the ones vector of size m. Here, the identity matrix signifies that the transportation process has the same origin and destination and the ones vector signifies that it applies to all transportation resources.

From these definitions of J_S and K_S, follows the definition of scleronomic transportation degrees of freedom.

Definition 11. Scleronomic Transportation Degrees of Freedom [3]: The set of independent transportation events $\mathcal{E}_S$ that completely defines the available transportation processes in a transportation system. Their number is given by:

$$DOF_S = \sigma(\mathcal{E}_S) = \sum_{u}^{\sigma(P)} \sum_{v}^{\sigma(R)} [J_S \ominus K_S](u,v) \qquad (9)$$

where the $A \ominus B$ operation is "boolean subtraction" which is equivalent to $A \cdot \bar{B}$. Note that the boolean "AND" $\cdot$ is equivalent to the hadamard product, and $\bar{B} = not(B)$. In matrix form, Equation 9 can be rewritten in terms of the Frobenius inner product [46]:

$$DOF_S = \langle J_S, \bar{K}_S \rangle_F = tr(J_S^T \bar{K}_S) \qquad (10)$$

The form of Equation 9 interestingly matches the form of the expression used for mechanical degrees of freedom. Furthermore, it allows the usage of the axiomatic design knowledge base for further detailed engineering design. Finally, the constraints matrix captures the potential for operational constraints like vehicle breakdowns, line closures, or road detours. As such, it allows a flexible expression of transportation system capabilities in the design and operational phases.

B. Rheonomic Transportation Degrees of Freedom

The previous subsection recalled the development of transportation scleronomic degrees of freedom as independent events. A transportation system, however, has constraints that introduce dependencies between events. This section addresses the transportation system's capabilities as sequences of events called strings and introduces rheonomic transportation degrees of freedom as a measure for their assessment [12].

Definition 12. Rheonomic Transportation Degrees of Freedom [3]: The set of independent transportation strings $\mathcal{Z}$ that completely describe the transportation system language.

In other words, the transportation system language $\mathcal{L}$ can be described equally well in terms of the Kleene closure [44] of the scleronomic and rheonomic transportation degrees of freedom.

$$\mathcal{L} = \mathcal{E}^* = \mathcal{Z}^* \qquad (11)$$

To begin, the transportation system capabilities are considered as strings of length 2 within the transportation system rheonomic knowledge base. Strings of longer length are addressed in Section IV.

Definition 13. Transportation System Rheonomic Knowledge Base [3], [14]: Given string $z_{w_1 w_2} = e_{u_1 v_1} e_{u_2 v_2} \in Z$ where events $e_{u_1 v_1}$ and $e_{u_2 v_2}$ are scleronomic transportation degrees of freedom, the transportation system rheonomic knowledge base is a binary square matrix J_R of size $\sigma(P)\sigma(R) \times$

$\sigma(P)\sigma(R)$ whose elements $J(w_1, w_2) \in \{0, 1\}$ are equal to one when string z_{w_1, w_2} exists.

It can be calculated straightforwardly by [3]

$$J_R = [J_S \cdot \bar{K}_S]^V [J_S \cdot \bar{K}_S]^{VT} \tag{12}$$

Here, the scleronomic transportation degrees of freedom are treated as a basis vector as would typically be done with mechanical degrees of freedom. J_R also strongly resembles an adjacency matrix where the scleronomic degrees of freedom are treated as nodes and are mutually connected.

As in the previous subsection, a constraints matrix K_R is introduced to describe the potential elimination of transportation system capabilities. Intuitively speaking, certain transportation events can follow one another, while others are not possible.

Definition 14. Transportation System Rheonomic Constraints Matrix [3], [14]: K_R of size $\sigma(P)\sigma(R) \times \sigma(P)\sigma(R)$ whose elements $K_R(w_1, w_2) \in \{0, 1\}$ are equal to one when a constraint eliminates string $z_{w_1 w_2}$ from the string set.

While K_S can equal zero, K_R has perpetual non-zero continuity constraints. In order for one degree of freedom to follow another, the destination of the former must be equivalent to the origin of the latter. Once again, this intuitive constraint can be described formally via the index conventions in Equations 3 and 6. Given two arbitrary transportation processes p_{u_1} and p_{u_2} which have origins and destination pairs $(b_{y_{11}}, b_{y_{12}})$ and $(b_{y_{21}}, b_{y_{22}})$ respectively, the perpetual non-zero continuity constraints requires that $y_{12} = y_{21}$. This may be expressed in a square matrix C_ρ of size $\sigma(P) \times \sigma(P)$ for all pairs of transportation processes [3]:

$$C_\rho(u_1, u_2) = \begin{cases} 1 & \text{if } \mod((u_1 - 1), \sigma(B)) = \\ & \qquad\qquad (u_2 - 1)/\sigma(B) \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

or equivalently [3]:

$$C_\rho = \mathbb{1}^{\sigma(B)} \otimes \left[\mathbf{I}^{\sigma(B)} \otimes \mathbb{1}^{\sigma(B)T} \right] \tag{14}$$

Here the term $\mathbb{1}^{\sigma(B)}$ physically signifies "any origin", the term $\mathbf{I}^{\sigma(B)}$ signifies "two equivalent stations", and the final term $\mathbb{1}^{\sigma(B)T}$ signifies "any destination". From this, the minimal transportation system rheonomic constraint matrix can be calculated straightforwardly [3].

$$K_R = \text{not} \left[[\mathbb{1}^{\sigma(R)}\mathbb{1}^{\sigma(R)T}] \otimes C_\rho \right] \tag{15}$$

Again the two $\mathbb{1}_{\sigma(R)}$ terms signify that any transportation resource can be used for either the first or the second transportation processes. As in its scleronomic counterpart, K_R may have additional constraints which may be captured manually or automatically depending on the current state of operation.

From these definitions of J_R and K_R follows the number of rheonomic transportation degrees of freedom using the same mathematical form of Equations 9 and 10 [3], [14]:

$$DOF_\rho = \sum_{w_1}^{\sigma(E_s)} \sum_{w_2}^{\sigma(E_s)} [J_R \ominus K_R](w_1, w_2) \tag{16}$$
$$= \langle J_R, \bar{K}_R \rangle_F = tr(J_R^T \bar{K}_R)$$

For the purposes of facilitating the next section and gaining further intuition, Equation 16 can be rewritten in a scalar form [13]. DOF_ρ

$$= \sum_{u_1}^{\sigma(P)} \sum_{u_2}^{\sigma(P)} \sum_{v_1}^{\sigma(R)} \sum_{v_2}^{\sigma(R)} \Big[[J_S \cdot \bar{K}_S](u_1, v_1) \qquad (17)$$
$$\cdot [J_S \cdot \bar{K}_S](u_2, v_2) \cdot \bar{C}_\rho(u_1, u_2) \Big]$$

Equation 17 views rheonomic degrees of freedom as a sequence of three binary conditions: one for each event in the string and one for their feasibility as a sequence.

This section has reused the axiomatic design large flexible system knowledge base to introduce the concept of scleronomic and rheonomic transportation degrees of freedom. These measures are used in the next section to enumerate the number of passenger itineraries.

IV. ENUMERATED ITINERARIES – PASSENGER DEGREES OF FREEDOM

As inspired by research in product degrees of freedom [13], [14], the passenger degrees of freedom measure takes advantage of the efforts in the previous section to measure the number of ways that a passenger in the transportation system may be transported from a desired origin to a final destination (i.e. the number of possible itineraries). The derivation rests on three definitions:

Definition 15. Passenger Event [3]: A single scleronomic transportation degree of freedom that permits a passenger's transport from a desired origin b_{y_1} to a desired destination b_{y_n}.

Definition 16. Passenger Itinerary [3]: A string of passenger events that permit the passenger's transport from a desired origin b_{y_1} to a desired destination b_{y_n}.

Definition 17. Passenger Degrees of Freedom (DOF_p) [3]: The number of passenger itinerary strings in the language $\mathcal{L}$ between a desired origin b_{y_1} to a desired destination b_{y_n}.

From these definitions, a straightforward derivation of the passenger degrees of freedom is to sum the itineraries consisting of 1 event, 2 events, up to the number of events n deemed impractical by the passenger [3].

$$DOF_p = \sum_i^n DOF_{pi} \tag{18}$$

The number of direct routes follows from Equation 10. Given an origin b_{y_1} and destination b_{y_2}, the transportation system scleronomic knowledge base row corresponding to transportation process p_u is selected out with an elementary basis vector e_u of appropriate size [3].

$$DOF_{p1} = \langle e_u^T J_S, e_u^T \bar{K}_S \rangle_F = tr((e_u^T J_S)^T (e_u^T \bar{K}_S)) \tag{19}$$

The number of two-leg routes is found by a similar strategy; this time by selecting the rheonomic transportation degrees of freedom that begin with the passenger origin station b_{y_1} and

end with the passenger destination b_{y_2}. Equation 8 is updated accordingly [3].

$$
\begin{aligned}
K_{S_{y_1}} &= K_S \oplus \text{not} \left[[e_{y_1}^{\sigma(B)} \otimes \mathbb{1}^{\sigma(B)}] \otimes [\mathbb{1}^{\sigma(R)}]^T \right] \\
K_{S_{y_2}} &= K_S \oplus \text{not} \left[[\mathbb{1}^{\sigma(B)} \otimes e_{y_2}^{\sigma(B)}] \otimes [\mathbb{1}^{\sigma(R)}]^T \right]
\end{aligned}
\tag{20}
$$

Here, the term $e_{y_1}^{\sigma(B)} \otimes \mathbb{1}^{\sigma(B)}$ signifies a transportation process beginning at y_1 and then going to any station while the term $\mathbb{1}^{\sigma(B)} \otimes e_{y_2}^{\sigma(B)}$ signifies a transportation process beginning at any station but ending at y_2. The $\mathbb{1}^{\sigma(R)}$ term signifies the application to all transportation resources. With this understanding, J_R is updated from Equation 12 accordingly [3].

$$
J_{R_{y_1 y_2}} = [J_S \cdot \bar{K}_{S_{y_1}}]^V [J_S \cdot \bar{K}_{S_{y_2}}]^{VT}
\tag{21}
$$

and the associated 2-event passenger degrees of freedom becomes [3]:

$$
DOF_{p2} = \langle J_{R_{y_1 y_2}}, \bar{K}_R \rangle_F = tr(J_{R_{y_1 y_2}}^T \bar{K}_R)
\tag{22}
$$

The number of n-event passenger itineraries is derived from the scalar form in Equation 17 where strings of the form $z = e_{u_1 v_1} e_{u_2 v_2}$ were considered. Extending this string to n events $z = e_{u_1 v_1} \dots e_{u_n v_n}$ yields the number of n-event rheonomic transportation degrees of freedom [3].

$$
DOF_{\rho n} = \sum_{u_1, \dots, u_n}^{\sigma(P)} \sum_{v_1, \dots, v_n}^{\sigma(R)} \left[\prod_x^{n-1} [J_S \cdot \bar{K}_S](u_x, v_x) \cdot \bar{C}_\rho(u_x, u_{x+1}) \right] \cdot [J_S \cdot \bar{K}_S](u_n, v_n)
\tag{23}
$$

This rather cumbersome scalar form based upon single events can be simplified by recalling that the product in Equation 16 is a square adjacency matrix A_R between scleronomic transportation degrees of freedom [3].

$$
A_R = J_R \cdot \bar{K}_R
\tag{24}
$$

Following the initial introduction to graph theory, where the nth power of an adjacency can be used to calculate the n-step paths through a network [8],

$$
DOF_{\rho n} = \sum_{w_1}^{\sigma(E_s)} \sum_{w_2}^{\sigma(E_s)} A_R^{n-1}(w_1, w_2)
\tag{25}
$$

Here, the $(n-1)$ power originates from the differences between the traditional formulation of the transportation network graph and that of the axiomatic design based approach. To fix the passenger itineraries specifically from the desired origin b_{y_1} to a desired destination b_{y_n}, Equation 25 becomes [3]

$$
DOF_{\rho n} = \sum_{w_1}^{\sigma(E_s)} \sum_{w_2}^{\sigma(E_s)} \left[A_{R_{y_1}} A_R^{n-3} A_{R_{y_2}} \right](w_1, w_2)
\tag{26}
$$

where [3]

$$
\begin{aligned}
A_{R_{y_1}} &= \left[[J_S \cdot \bar{K}_{S_{y_1}}]^V [J_S \cdot \bar{K}_S]^{VT} \right] \cdot \bar{K}_R \\
A_{R_{y_2}} &= \left[[J_S \cdot \bar{K}_S]^V [J_S \cdot \bar{K}_{S_{y_2}}]^{VT} \right] \cdot \bar{K}_R
\end{aligned}
\tag{27}
$$

In this section, passengers were modeled in terms of sequences, which allowed for the enumeration of their itineraries in a measure called passenger degree of freedom measures. All measures continued to exhibit the same three common elements found in mechanical degrees of freedom: discrete events captured in axiomatic design knowledge bases, constraint matrices, and a boolean difference of these two matrices. The transportation degrees of freedom broadly measure "reconfiguration potential". The scleronomic transportation degree of freedom measures provide a quantitative description of which transportation capabilities exist in the system and potentially how they can be changed. Mathematically, it can be described as a reconfiguration process $\mathbf{R}$ [47]:

$$
\mathbf{R}(J_S, K_S) \rightarrow \mathbf{R}(J_S', K_S')
\tag{28}
$$

The rheonomic transportation degree of freedom measures provide a quantitative description of how transportation capabilities can be combined into sequences. In either case, these measures describe the impact of the desired set of reconfigurations on the system capabilities. Mathematically, it can be described by the transformation [47]:

$$
\mathbf{R}(J_R, K_R) \rightarrow \mathbf{R}(J_R', K_R')
\tag{29}
$$

V. ILLUSTRATIVE EXAMPLE: MEXICO CITY PUBLIC TRANSPORTATION SYSTEM
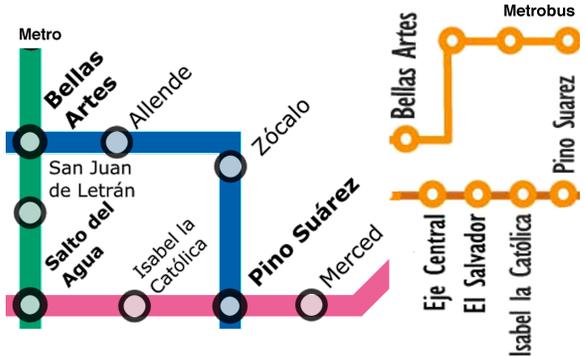

Fig. 1. Metro & Metrobus stations covered in Illustrative Example [49], [50]

To demonstrate the application of the passenger DOF measures, real data from the Mexico City Public Transportation System is taken as an illustrative example. This system is one of the largest of its kind in the world and includes various modes of transportation, such as light rail, the bus network, the Metro and Metrobus. It serves a population of approximately 25 million and has over 300 stations [48]. In order to facilitate the illustration and development of intuition, the system boundary is narrowed to a few square blocks around the City Center as shown in Figure 1. It includes the following 10 stations in index order: Merced, Pino Suarez, Isabel la Catolica, Salto del Agua, San Juan de Letran, Bellas Artes, Allende, Zocalo, Eje Central, El Salvador. Therefore, $\sigma(B) = 10, \sigma(P) = \sigma^2(B) = 100$. Further data on the Mexico City transportation system may be found on their website [49], [50].

Continuing with the numerical illustration, the set of resources is defined as R={Pink Line, Green Line, Blue Line, Metrobus}. $\sigma(R) = 4$. By Definition 9, the system knowledge base is formed by inspection. Equation 8 is then used to

calculate the transportation system scleronomic constraints matrix. Figure 2 shows the two matrices as monochrome images with each dark pixel representing a 1 element in the respective matrix. Equation 9 then yields $DOF_S = 21$ after 14 null processes have been eliminated from the event set by K_S. These 21 scleronomic transportation degrees of freedom represent the smallest set of discrete events that must be enabled by automation or human operation to allow full functionality of the transportation system. From the perspective of automation deployment, a specific program can be deployed for each of these DOFs. Next, J_R and K_R are calculated directly from Equations 12 and 15 respectively. Equation 16 then yield $DOF_\rho = 52$ which may be verified manually by brute-force inspection.
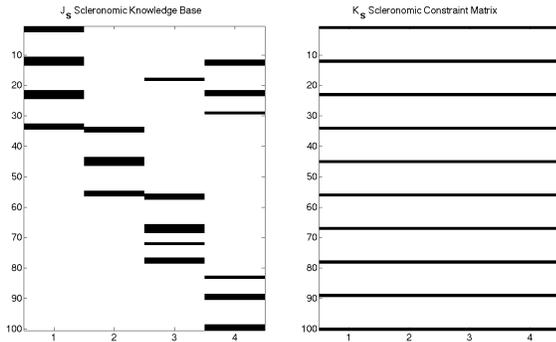

Fig. 2. Scleronomic Knowledge Base J_S and Constraint Matrix K_S

Now consider a passenger who wishes to travel from b_2 =Pino Suarez to b_6 =Bellas Artes. The equations in Section IV are applied sequentially and give the following results $DOF_{p1} = 0$, $DOF_{p2} = 0$, $DOF_{p3} = 1$, and $DOF_{p4} = 2$. There are no itineraries between the desired origin and destination in 1 or 2 stops. There is one such itinerary in three stops along the blue line. There is a 4-stop itinerary between the pink and green line, and another that begins on the metro bus and then continues on the pink and green lines. Further illustrative examples of the transportation degree of freedom approach may be found in [12], [13], [29].

This numerical example of medium complexity does suggest a number of interesting points about the potential application of the method. First of all, as transportation degrees of freedom represent the minimal set of discrete-events to describe the system, they can be used directly in the formulation of intelligent control and automation solutions. Another main advantage is that the axiomatic design approach allows the set of resources to be viewed in whichever level of aggregation is deemed most appropriate for design, analysis and control. The set of resource could have equivalently defined as R={Metro, MetroBus} so that each transportation mode is organized by its respective transportation operator. More detailed design of the transportation system can instantiate the resources to individual trains and buses. Another interesting point is the reconfiguration time-scale. Here, the knowledge base is viewed as constant over an hourly time scale. However, later on in the night certain metro or bus lines may become closed or made to have express-only service. The knowledge bases and constraints matrices would evolve accordingly. Finally, these

methods are extensible to full size transportation systems. In such an instance, sparse matrix methods can be applied to take advantage of the sparsity of the knowledge bases and constraint matrices [51].

VI. DISCUSSION: RELATIONSHIPS OF AXIOMATIC DESIGN TO GRAPH THEORY

Having discussed the axiomatic design based transportation degree of freedom measures, and demonstrated their application to real-world example, this section discusses the relation between the axiomatic design knowledge bases and the traditional graph based models in transportation systems. Fundamentally speaking, both approaches rely on graph theory but utilize different definitions of nodes and arcs to achieve complementary results.

A. Scleronomic Transportation DOF

A first observation is that the transportation system scleronomic knowledge base can be calculated from the traditional graph theory based approach. Given a digraph $D = (B, P)$, the associated incidence matrix M may be used to calculate the scleronomic knowledge base J_S.

$$\bigvee_{i}^{\sigma(B)} \mod M(i,j) + (\mathbf{I}^{\sigma(P)})^V = J_S(j,1) \qquad (30)$$

where the vectorized identity matrix is used to account for the null processes.

Note, here, that the traditional graph theoretic definition lacks in four ways. First, it does not explicitly describe the different modes of transport R. Multiple modes are often treated with an edge color for each mode [9]. This requires a different incidence matrix for each transport mode. The transportation system scleronomic knowledge base, therefore, more easily lends itself to the analysis of multi-modal systems. Second, although the null-processes were not given extensive attention in this work, they do fulfill the important transportation system functions of parking and loading/unloading. From the perspective of reconfigurable engineering design, the inclusion of the null processes in the knowledge base allows for these functions to be explicitly considered within the system's holistic function. Recall that previous transportation system axiomatic design applications have specifically addressed these "null-processes" in the form of road intersections [33]–[36], airport terminals [37], and shipping ports [38]–[40]. These "null-processes" may gain further importance with electric vehicles and their need for well-rationalized charging station design and placement [52]. Extensions of this work has already shown the importance of these "null-processes" in reconfigurable manufacturing systems [12], [13], [29]. It is not clear, how such analytical functionality can be achieved with the traditional transportation system graph definition. Thirdly, within axiomatic design, functional and physical decomposition of the knowledge base [11] can be used directly for further detailed design. Examples of such a design practice include modular buildings [53], [54] and intelligent manufacturing systems [12]. Finally, the use of a scleronomic constraints matrix easily allows the description of

faulted or intentionally-offline transportation system functionality. From the perspective of reconfigurable system design, these constraints can potentially be removed or reconfigured for alternate modes of operation. For example, the switches in a rail network move a constraint from one rail fork to another.

Another interesting relationship between the axiomatic design based approach and graph theory is that the scleronomic knowledge base forms a bipartite graph between the set of processes and resources. Consider a bipartite graph $G = \{P \cup R, E\}$. The associated adjacency matrix A can be calculated from the scleronomic knowledge base J_S.

$$A = \left[\begin{array}{cc} \mathbf{0} & J_S \\ J_S^T & \mathbf{0} \end{array} \right] \qquad (31)$$

The usage of this adjacency matrix only serves to demonstrate the relationship clearly but is both redundant and oversized.

B. Rheonomic Transportation DOF

In a similar way, the rheonomic knowledge base can also be viewed as graph theoretic. It takes J_S^V is a basis vector of transportation system degrees of freedom as is the case of mechanical degrees of freedom. Then, it uses these individual degrees of freedom as a graph's node. More formally, consider the graph $G = \{\mathcal{E}_S, \mathcal{E}\}$, then $J_R = A$. This result was ultimately used to derive the number of passenger itineraries.

VII. DISCUSSION: RECONFIGURABILITY APPLICATIONS OF AXIOMATIC DESIGN

Axiomatic design has proven a powerful tool to measure transportation degrees of freedom as a measure of reconfiguration potential. This section discusses three classes of applications for these developments: reconfigurable operations, reconfigurable planning, and reconfigurability valuation.

A. Reconfigurable Operations

The concept of transportation degrees of freedom can be applied to achieve reconfigurable transportation system operations when the knowledge base and constraint matrices are taken over a short but regular time interval i.e. one hour. In such a case, a reconfiguration process can be said to occur from one hour to the next. For example, not all bus and metro lines are in service at all times in the day. Their periods of non-operation can be captured within a constraints matrix. Similarly, some routes become "non-stop" or "express" during certain times in the day.

These observations suggest that there exist many types of constraints that limit the reconfiguration potential of the transportation system. For example, many rail and metro lines use rigid supervisory execution control [55]. One can easily conceive code that pushes trains without choice down a dedicated line. The resulting transportation system language would be $\mathcal{L} = e_{u_1v_1}e_{u_2r_1}e_{u_3r_1}$ when it could have been written to support the language $\mathcal{L} = \{e_{u_1v_1}e_{u_2r_1}e_{u_3r_1}\}^*$. In essence, railway operators that engage in active real-time switching sequences can be viewed as making real-time reconfigurations, or eliminating scleronomic and rheonomic constraints all together. Fixed public transportation system schedules are another example of inflexible operations. Bus and trains leave at a fixed time irrespective of existing traffic

conditions or vehicle breakdowns elsewhere in the system. Real-time transportation scheduling algorithms represent a key enabling technology for reconfigurable operations in the face of disturbances and shocks to the system.

B. Planning

The concept of transportation degrees of freedom can also be applied to long-term planning decisions. In the medium term, the schedules generated by transportation system operators represent a planning activity of which transportation system resources are going to be used to realize which transportation system processes. In the long term, the expansion of a transportation system network represents an expansion of the system knowledge base to include new transportation processes (i.e. rows in the knowledge base) and new transportation resources/modes (i.e. columns in the knowledge base).

Returning to the Mexico city system as an example, the reader is taken back to the late 1990's, before the Metrobus was developed. Back then, typical city buses covered the streets of the downtown area, contributing to what was already the most heavily-congested traffic area in the city. Even worse, the service was lackluster due to the long trip times between locations that were oftentimes reached faster on foot rather than by bus. The Metro, known then for being crowded to the point of being uncomfortable and a safety hazard, was avoided by many passengers. A decision was made to expand introduce the Metrobus system on surface streets on dedicated median lanes with virtually no traffic congestion. Mathematically, the reconfiguration process mentioned in Equation 28 corresponds to the addition of the fourth column of scleronomic knowledge base and constraint matrix. Table I shows the transportation degrees of freedom expanding dramatically before and after 2005.

TABLE I
TRANSPORTATION SYSTEM DEGREES OF FREEDOM

	Before 2005	After 2005
DOF_S	16	22
DOF_ρ	34	58

C. The value of reconfigurability

The concept of transportation degrees of freedom as a measure of reconfiguration potential draws out questions as to the value of this reconfigurability. To this end, it is important to recognize that each transportation degree of freedom can be associated with tangible measures that figure prominently in ROI and cost/benefit decisions. In operations, each degree of freedom is associated with a passenger capacity and hence a revenue. Alternatively, it can be associated with a time of execution, energy consumption, greenhouse gas emissions, operating costs, and externalities. Furthermore, one can measure the ease of a reconfiguration process and value it in terms of time or monetary cost [47]. In such a strategy, it becomes possible to value reconfigurability as an operations-stage life cycle property. In planning decisions, each degree of freedom can be associated with not just an expected capacity and revenue, but also the required investment to make the degree of freedom possible. Similarly, such an approach can be used to model future energy consumption and greenhouse

gas emissions from a perspective of technical planning rather than macroeconomic development.

VIII. Conclusions & Future Work

This paper has developed a set of system axiomatic design based measures called transportation system degrees of freedom to enumerate passenger itineraries. The work rests firmly on the foundation of previous work in the field of reconfigurable manufacturing systems. Specifically, this includes axiomatic design for large flexible systems, graph theory, and an analogy between the behavior of mechanical systems and the architecture of transportations systems. These measures were developed to measure the sequence-independent and sequence dependent capabilities of the transportation system. The scleronomic measure was useful in describing individual process/resource combinations coming online and offline. Meanwhile, the rheonomic measure described the continuity limitations when pairs of process/resource combinations were put into a sequence. In practice, the rheonomic measures could be used to identify rigidities in intelligent transportation systems that prevent the full reconfiguration potential of the physical transportation system. These measures were later used to enumerate the number of passenger itineraries between an origin station and a destination. These measures were ultimately demonstrated on a small subsection of the Mexico City transportation system. As a set, these measures give a thorough understanding of the potential for reconfiguration as individual events or their their sequences.

The axiomatic design approach was related to the traditional graph theory model of a transportation system. While the two approaches are complementary, the axiomatic design does present four clear advantages. 1.) Multiple modes of transport are explicitly modeled in a single binary matrix 2.) Null-transportation processes where no motion occurs are also explicitly modeled as functions which facilitate the engineering design of transportation system nodes like road intersections, airport terminals, shipping ports, electric vehicle charging, and product manufacture. 3.) The axiomatic design knowledge bases are firmly routed in the language of engineering design where function is directly allocated to form. Detailed engineering design of both physical infrastructure as well as its supporting control, automation and IT systems can proceed straightforwardly from the decomposition of function and form. 4.) Finally, the use of constraints matrices allows for the easy description of faulted or intentionally-offline transportation system functionality.

In the future, the authors seek to extend this work to study the resilience of transportation systems. One key question is the decision-making criteria that cause for the system to be reconfigured; particularly in the operation time scale. Such criteria can include measured or forecasted shifts in demand over the course of a normal day or sudden shocks such as major sporting events or natural disasters. Intuitively speaking resilient system are likely to have greater reconfiguration potential. Such work can shed further insight on the value of reconfigurability in response to scenarios requiring resilient operation. The authors especially wish to study transportation systems where the stations, system functionality and modes of

transport have a heterogeneous nature; thus fully utilizing the richness of the modeling framework.

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