

# Real-Time Economic Dispatch for the Supply Side of the Energy-Water Nexus

Apoorva Santhosh, Amro M. Farid, Kamal Youcef-Toumi

## Abstract

Clean energy and water are two essential resources that any society must securely deliver. Their usage raises sustainability issues and questions of nations' resilience in face of global changes and mega-trends such as population growth, global climate change, and economic growth. Traditionally, the infrastructure systems that deliver these precious commodities, the water distribution and power transmission networks are thought of as separate, uncoupled systems. However, in reality, they are very much coupled in what is commonly known as the energy-water nexus. Although this subject has recently caught the attention of numerous policy and regulatory agencies, rarely is it holistically addressed in terms of an integrated engineering system for its management, planning, and regulation as an interdisciplinary concern. This work specifically fills this gap by addressing the supply side of this integrated engineering system. Specifically, it develops the multi-plant real-time simultaneous economic dispatch of power and water. While significant background literature has addressed traditional power dispatch, and the emerging co-dispatch of power and heat, as of now there does not exist a parameterized model for the optimized dispatch of power and water for multiple power, water, and coproduction facilities. The work presents such a model where production costs are minimized subject to capacity, demand and process constraints. It is demonstrated on an illustrative example of modest size. Interesting results were observed suggesting that the coproduction minimum capacity limits and process constraints can lead to scenarios where cheaper single product plants can be crowded out of the dispatch. The program provides a systematic method of achieving optimal results and can serve as a basis for set-points upon which individual plants can implement their optimal control. In so doing, it makes a supply-side contribution to the ongoing grand-challenge of improving the sustainability of the energy-water nexus.

## NOMENCLATURE

$\bar{r}_k$	Max. Power-Water production ratio of the $k^{th}$ coproduction plant
$\bar{X}_{ck}$	Max. capacity limit of the $k^{th}$ coproduction plant
$\bar{X}_{pi}$	Max. capacity limit of the $i^{th}$ power plant
$\bar{x}_{pi}$	Max. power capacity limit of the $i^{th}$ power plant
$\bar{X}_{wj}$	Max. capacity limit of the $j^{th}$ water plant
$r_k$	Min. Power-Water production ratio of the $k^{th}$ coproduction plant
$X_{ck}$	Min. capacity limit of the $k^{th}$ coproduction plant
$X_{pi}$	Min. capacity limit of the $i^{th}$ power plant
$x_{pi}$	Min. power capacity limit of the $i^{th}$ power plant
$X_{wj}$	Min. capacity limit of the $j^{th}$ water plant
$C_G$	Production cost function
$C_{ck}$	Cost function for $k^{th}$ coproduction plant
$C_{pi}$	Cost function for $i^{th}$ power generation plant
$C_{wj}$	Cost function for $j^{th}$ water production plant
$n_c$	Number of coproduction plants
$n_p$	Number of power generation plants

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$n_w$	Number of water production plants
$\mathcal{K}_{ck}$	Linear prod. cost function coeff. of $k^{th}$ coproduction plant
$\mathcal{K}_{pi}$	Linear prod. cost function coeff. of $i^{th}$ power plant
$\mathcal{K}_{wj}$	Linear prod. cost function coeff. of $j^{th}$ power plant
$A_{ck}$	Quadratic prod. cost function coeff. of $k^{th}$ coproduction plant
$A_{pi}$	Quadratic prod. cost function coeff. of $i^{th}$ power plant
$A_{wj}$	Quadratic prod. cost function coeff. of $j^{th}$ power plant
$B_{ck}$	Linear prod. cost function coeff. of $k^{th}$ coproduction plant
$B_{pi}$	Linear prod. cost function coeff. of $i^{th}$ power plant
$B_{wj}$	Linear prod. cost function coeff. of $j^{th}$ power plant
$D$	Demand vector
$D_p$	Electrical power demand
$D_w$	Water demand
$X_{ck}$	Decision vector for the $k^{th}$ coproduction plant
$x_{cpk}$	Power generated at the $k^{th}$ coproduction plant
$x_{cwk}$	Water produced at the $k^{th}$ coproduction plant
$X_{pi}$	Decision vector for the $i^{th}$ power plant
$x_{pi}$	Power generated at the $i^{th}$ power plant
$X_{wj}$	Decision vector for the $j^{th}$ water plant
$x_{wj}$	Water produced by the $j^{th}$ water plant

## I. INTRODUCTION

### A. Motivation

Clean energy and water are two essential resources that any society must securely deliver [1] in order to develop sustainably; i.e. meet its economic, social and environmental goals [1]–[4]. In the case of energy, the overuse of conventional resources has raised concerns over global climate change, smog and acid rain collectively [5]. Consequently, research into clean renewable energy resources and energy efficiency methods has increased. Similarly, potable water is another vital resource for survival and development. Increased water use has grown substantially in recent years; tracking strongly with energy use and economic development and leading to depleted water tables in many geographic regions [6]. Therefore, there is an important need to optimize both energy and water resources in light of quickly depleting global energy resources, increased concern about national security, and the need for sustainable economic growth.

Energy and water usage also raises sustainability issues and questions of nations resilience in the face of global changes and mega-trends [2], [6]. Developing nations, in particular, project strong water and electricity demand growth driven by population growth. Economic growth and the associated improvements in individual lifestyles also imply a subsequent intensification in the per capita demand of water and electrical energy. Furthermore, the hot and arid climates found in many developing nations amplify the coupling of water and energy use. As a result, typical water deficits may intensify into aggravated water scarcity. Global climate change projections foresee further distortions in the availability and consumption of fresh water [7]. The nations that take early planning measures in their infrastructure development are likely to be in a better position to mitigate energy and water usage impacts.

### B. Scope

Traditionally water distribution and power transmission networks are thought of as separate uncoupled infrastructure systems. And yet, these two essential resources are intrinsically coupled in that the production, distribution and consumption of one often requires the other [8]. This interlinked meta-system is often called the energy-water nexus and is defined here as:

**Definition 1.** Energy-Water Nexus [9]–[12]: A system-of-systems composed of one infrastructure system with the artifacts necessary to describe a full energy value chain and another infrastructure system with the artifacts necessary to describe a full water value chain.

TABLE I  
SUPPLY & DEMAND SIDE ENERGY-WATER NEXUS COUPLINGS

	Power Supply	Power Demand
<b>Water Supply</b>	Co-generation: <ul style="list-style-type: none"> <li>• Thermal Desalination</li> <li>• Hydroelectric</li> </ul>	<ul style="list-style-type: none"> <li>• Pumped Water</li> <li>• Water Distribution</li> <li>• Wastewater Recycling</li> </ul>
<b>Water Demand</b>	Thermal-Power Generation Facilities	Residential, Commercial, & Industrial Use of Electric Heating & Cooling of Water

Table I shows the couplings between the energy and water systems. The greatest attention has been given to the cross-interactions of energy supply to water demand or vice versa. Many empirical methods and studies have attempted to quantitatively assess the water consumption requirements and impacts of thermal power generation facilities [13]–[21]. Similarly, research is underway to improve technologies that would diminish this impact [22], [23]. Another cross-coupling is the electrical pumping energy required to produce and dispatch potable water [24]–[27]. As the subject of this paper, coproduction facilities like hydroelectric and thermal desalination plants [28] couple the respective supply sides of energy and water [29]. Finally, the residential, commercial, and industrial use of electric heating and cooling for water consumption presents a major coupling on the demand side of both systems [30].

Although the energy-water nexus has recently caught the attention of numerous policy and regulatory agencies [13], [24], [30]–[35], rarely [36]–[38], is it holistically addressed in terms of an integrated engineering system framework for its management, planning and regulation as an interdisciplinary concern. Recent research into smart (power) grid activities implicitly require a rebalancing of power generation technology portfolio [39]. Similarly, smart (water) grid activities implicitly require a rebalancing of the water supply technologies be they desalination, groundwater pumping or water recycling [40], [41]. Paradigmatically, as well as technologically, there is a great potential for convergence of these work streams. Recently, the energy-water nexus has been modeled as an integrated engineering system [9], [10] and initial work for holistic quantified planning is reported [11], [12].

This paper restricts its scope to the real-time economic dispatch of the supply-side of the engineered electricity and water systems. This includes the couplings manifested by the operations management of hydroelectric and thermal desalination facilities. Hydro-electric facilities have a hydro-power production function that ties the output power to the spillage [42]–[45]. Meanwhile, thermal desalination facilities require a steam balance that couples the heat by-product of power generation to the production of potable water [46]–[48]. Therefore, in this approach, the optimal mix of produced water and power is determined by how these plants operate within the greater context of water and power real-time economic dispatch. The term “real-time” economic dispatch is borrowed from power system market operation to refer to a time scale that ranges from 5 minutes to 1 hour.

### C. Relevance

The optimization of the supply side coupling is of greatest interest in the Gulf Cooperation Council (GCC) countries. The hot and arid climates found in the GCC cause a heavy reliance on desalination technology to alleviate the scarcity of potable ground water. The additional reliance on climate-controlled buildings further exacerbates power dispatch with sharp peak loads. Fortunately, most GCC nations operate their water and power utilities within a single organization and therefore the optimization program presented in this work is of direct industrial applicability [49]. Similar combined water and power utilities may be found in other regions of the world. Furthermore, the presence of a cooptimization program can highlight potential efficiencies if separated power and water utilities were to coordinate their activities. The cost-optimization program presented in this work is first intended for the GCC region which works under regulated frameworks. Ultimately, this work can serve in the development of an integrated energy-water market not unlike deregulated energy markets found in European and North American nations.

#### D. Contribution

This paper constitutes the first part in a three part series in which traditional power system optimization techniques are generalized from power-only markets to the energy-water nexus supply side. The first of these optimization programs is the real-time economic dispatch [50], [51]. It is further developed here. Its optimization program is compact so as to limit computational intensity and facilitate implementation on a time scale between 5 minutes and 1 hour. This work is complemented by an expanded program that includes ramping rates and the storage of both water and energy [52], [53]. It closely resembles a power system unit commitment or “look-ahead dispatch”. Finally, the third part in the series introduces the effect of power and water network capacity limits [54]. It closely resemble a power system DC optimal power flow.

#### E. Paper Outline

The remainder of the paper develops in six sections. Section 2 highlights aspects of the background literature to power-water co-optimization. Specifically, the power economic dispatch problem formulation is introduced as a prerequisite single-product optimization and then the dual-product co-optimization literature is reviewed. The paper then presents the modeling methodology for the co-optimization of power and water and then proceeds to explain the simulation methodology in Section 4. Section 5 presents the results for an example system with 8 plants under three different operating scenarios: uncoupled, coupled-inflexible, coupled-flexible. The paper concludes in Section 6. Given the inter-disciplinary nature of the work, a large number of both context and foundational literature has been included.

## II. BACKGROUND LITERATURE

This section highlights aspects of the background literature for power-water co-optimization. First, power and water economic optimization literature are introduced as prerequisite single product optimizations. Next, the literature on dual product optimization for power and heat is reviewed. The section concludes with the mention of efforts to co-optimize power and water.

### A. Real-Time Economic Dispatch of Power - Problem Formulation

Economic dispatch [55] is the process of allocating the generation of power in a manner so as to minimize the cost of production of the needed power by encouraging the use of cheap fuels and/or most efficient plants. This is an area which has been developed extensively both academically and industrially [56], [57]. One of the most important advantages of the economic dispatch problem is that it allows for all types of generating plants to be treated equally despite their individual physical characteristics and constraints. It does so by focusing on creating a generalised algorithm with a uniform cost function and constraints in order to effectively handle different types of plants. Typically, the cost function is taken to be of quadratic form, however, different approaches also consider the cost to be linear, piecewise linear, and in certain cases of higher polynomials as needed [56], [58], [59].

Industrially speaking, the economic dispatch of power is usually handled in stages of increasing frequency rather than in a single pass optimization. The first stage is called the day-ahead energy market and uses a class of optimization program called unit commitment. It serves to to commit resources, and set approximate generation levels and prices for the 24 hours of the following day based upon demand bids, generation offers, and scheduled bilateral transactions [60]–[71]. The next stage in economic dispatch is the real time spot market [72]–[77]. In this real-time economic dispatch, electricity generation and prices are calculated at intervals between 5 minutes and 1 hour based upon actual grid operating conditions. This paper may be viewed as a development of a real-time economic dispatch for both power and water. The two stages applied together allow the dispatch to correct for errors in the demand forecast while respecting the startup and ramping capabilities of the generation facilities [78], [79]. Finally, the optimal power flow problem considers the power transmission constraints within the network [80]–[87].

The implementation of economic dispatch also varies widely depending upon the regulatory structure. They are broadly classified as regulated and unregulated. In the former, although the utility may purchase from a neighboring utility, it is vertically integrated and therefore primarily responsible for its own generation, transmission and distribution of power to the customers in its area. In the deregulated supply system, generation and distribution are uncoupled and customers are no longer bound to one utility but are free to purchase from any suppliers on the

grid. Purchasing of power is done via the electricity market mechanisms and transmission scheduling is conducted by the independent system operator [88]–[90]. Comprehensive treatments of deregulated electricity markets can be found in [91]–[95], while the literature also provides specialized treatments in deregulated price forecasting, dispatching and bidding simulation [96]–[98]. It is generally agreed that deregulated power systems provide greater market incentives for generation capacity investments and as a result allow for a smoother transition into newer technologies and greater flexibility [99].

From a purely mathematical point of view, real-time economic dispatch can be considered the most basic algorithm to deliver power in an economic fashion [55]. It minimizes the total power generation cost subject to the capacity limits of the power plants and the balance of power generation and demand under the lossless transmission assumption. It can be presented equivalently as either a primal or dual problem depending on the implementation in a regulated or deregulated market [57]. Formally, the primal problem is:

$$\min C_G(x_{pi}) = \sum_{i=1}^{n_p} C_{pi}(x_{pi}) \quad (1)$$

$$s.t. \sum_{i=1}^{n_p} (x_{pi}) = D_p \quad (2)$$

$$\underline{x}_{pi} \leq x_{pi} \leq \bar{x}_{pi} \quad (3)$$

### B. Real-Time Economic Dispatch of Municipal Water

In contrast to the well developed regulated and deregulated markets within the electrical infrastructure, the economic dispatch of water has achieved neither a comparable intellectual consensus nor widespread industrial adoption. In their recent review, Chong & Sunding outline the broad classes of objections to water markets and trading [100]. That said, the Dublin Principles arising from the 1992 International Conference on Water and Environment state [101]: “Managing water as an economic good is an important way of achieving efficient and equitable use, and of encouraging conservation and protection of water resources.” A number of different economic approaches to water management have emerged including general equilibrium models, cost-benefit analyses, agent-based models, and hydro-economic models [100].

The broader class of hydro-economic models are of greatest relevance here in that they can be considered to incorporate the real-time economic dispatch of water. Hydro-economic models typically use optimization over discrete time-steps to simulate the water resource management of a region. As the name suggests, they combine an economic model in the form of a monetization function of the water’s value with algebraic constraints that describe the engineering physics of the water flows in the region [100].

When applied to municipal water, the monetization often reflects the utility’s variable costs for water treatment and pumping. The underlying engineering model depends on the intended application but often includes state-space equations of the hydrodynamics, a connectivity matrix to represent the hydraulic network, and capacity limits on the reservoir sizes and releases [102]. The temporal resolution can range from hours to years and the spatial resolution can range from a household to groups of countries [100], [102]. Interestingly, the academic literature concerning the optimal pumping control problem for multi-reservoir systems has drawn extensively from techniques as varied as linear and nonlinear programming, stochastic optimization, dynamic programming and discrete-time optimal control [102]–[112]. As in power systems, multi-stage optimizations in which medium-term operations are cascaded into hourly optimizations have also been reported [113], [114]. These may be further cascaded into real-time reservoir control systems [115]–[117]. In all, there exists a rich academic literature to address the various types of economic dispatch of municipal water.

Nevertheless, these academic contributions have not necessarily translated to industrial implementation [100], [102], [103]. Significant anecdotal evidence has been reported within the literature over several decades suggesting a lack of industrial confidence in the applicability of the optimization models, combined with poorly designed incentives to improve operational efficiency [100], [102], [103], [118]. That said, the dozen or so successful real-time optimal pumping control implementations have relied on decision support systems and SCADA as enabling technologies. The continued development of these technologies in combination with increasing water scarcity pressures are likely to influence greater adoption of real-time economic dispatch of municipal water [102], [103].

### C. Co-Dispatch of Power and Heat

The consistent application of economic dispatch of power over decades has provided a market signal to not just reduce costs but also invest into more energy efficient technologies. In this regard, combined cycle power plants were systematically favored over single cycle facilities. Furthermore, facilities that cogenerated power and heat could demonstrate even higher efficiencies by using heat as a valued product for nearby industrial sectors such as food processing, chemical production, and district heating [119]–[121]. The resulting efficiency gains also bring about cost savings, reduced air pollution and greenhouse gas emissions, increased power reliability and quality, reduced grid congestion and avoided distribution losses [122]. Many policy-makers, particularly in Northern Europe, further supported dual product facilities through regulatory development [123]. Nevertheless, the technical and economic rationalization of a cogeneration solution often depended on the challenging conditions of having a consistently available, dedicated and co-located heat consumer [124] - often in the form of a contentiously negotiated [125], [126] long-term contract [127]. Naturally, some have argued to ease these restrictions on heat and power – as dual products – with a more dynamic treatment [128].

To that effect, a power-heat economic dispatch approach has been applied within the literature. Typically, it creates a single objective function for coproduction plants that is dependent on the amount of power and heat produced. Constraints are then added to set up limits for both power and heat capacities. These limits usually define a feasible region in which the cogeneration plant can operate with respect to power and heat produced [129]–[133].

### D. Co-optimization of Power and Water

Generally speaking, research on co-optimization of power and water has focused on the optimization of one particular plant and hence does not provide an extensible and general optimization formulation. For example, some focus on optimized planning and design rather than operations [134]–[136]. Still others find methods of cost allocation [137]. Finally, one author directly addresses the economic dispatch of a single MSF desalination facility composed of a number of sub-units but neither generalizes the formulation nor applies it to all the water and production units in the water and power grids [138]. In contrast, a number of robust optimization methods have been recently applied to hydrothermal systems [42], [139]–[144], but they do not specifically include the potable water demand from the water utility. To the authors' knowledge, there does not exist a parametrized model for the optimization of multiple co-generation plants in conjunction with pure power and water plants with no such assumptions of cost splitting. There exists a need for a model that allows for all three plants to be treated with equity. While similar techniques have been used for power-heat cogeneration, it has not been extensively explored in power water co-optimization and may serve as the basis of set point determination for single-plant optimization formulations.

## III. MODELING METHODOLOGY

This section describes the modeling methodology for the formulation of an optimization program to simultaneously dispatch power and water. Subsection 3.1 provides a conceptual model of the system of interest. Subsection 3.2 then recounts the optimization program first presented in [50] whose goal is to provide dispatch setpoints that individual facilities can use for single-plant target optimizations. Recalling the discussion of Section 1.4, the ultimate goal of this optimization program is to mimic the role of a real-time power market but instead for the supply-side of the energy-water nexus. Therefore, efforts are made to keep the optimization program as compact as possible to limit its computational intensity and facilitate its implementation on a time scale between 5 minutes and 1 hour.

### A. Conceptual Model

Figure 1 provides a graphical representation of the conceptual model that serves as the basis for the development of the optimization program. It consists of an integrated power & water utility that is interested in simultaneously serving an electrical power demand as well as a potable water demand. The respective grids are modeled as single nodes. The utility dispatches power, water, and coproduction facilities may be independent or vertically integrated. The power facilities may be taken as dispatchable thermal power generation plants. The coproduction facilities may be either hydroelectric or thermal desalination. They couple the respective grids by virtue of their production cost

functions and their process constraints. The water plant may be a ground or surface pumping station or a reverse osmosis desalination plant all with the necessary treatment capabilities to ensure potable water. The conceptual model and associated optimization program do not preclude a wastewater treatment plant from recycling water back into the potable water distribution network, although this remains an infrequent use case.

The choice of three types of facilities includes two inherent assumptions. The optimization programs in [52], [53] explicitly considered electrical energy and water storage facilities. Here, it is assumed that electrical energy storage facilities are of such a small penetration that their contribution can be neglected. In contrast, water utilities maintain water storage reservoirs whose volume capacity is often greater than many days worth of maximum flow release. Therefore, within the time scale of 5 minutes to 1 hour, it is highly unlikely that a storage facility will run up against its volume capacity limit. As a result, it is not essential to include the storage level of water storage facilities over this time scale. Instead, the set of water storage facilities may be included directly within the set of water production facilities. These two assumptions bring about a significant computational benefit in that the optimization time blocks can now be entirely decoupled, and a separate optimization can be run as each time block comes in “real-time”.

The power generation facilities require a fuel source whose cost serves as the basis of the production cost function. The same holds true for the coproduction facilities based upon thermal desalination.

Each water and coproduction facility is assumed to draw from its own independent water source. The model also applies to a single aggregate water source; as in the practical case of the Persian Gulf serving all desalination facilities in the U.A.E. Hydrologically speaking, the water sources are assumed to be able to support the maximum water flow capacities of the water production facilities that they serve. The power and water demands are measured net of any power and water requirements to the dispatched facilities and are ultimately delivered to the utility’s power and water customers.

## B. Problem Formulation

The formulation of the power-water co-optimization first proposed in [50] is as follows. Minimize the production cost objective function  $C_G$  with respect to the quantity of power generated by a power plant  $x_{pi}$ , water produced by a water plant  $x_{wj}$ , the power generated by a coproduction plant  $x_{cpk}$  and water produced by a coproduction plant  $x_{cwk}$  in the three types of plants:  $i, j, k$  power, water, and coproduction respectively. The following notations are introduced to vectorize the formulation:  $X_{pi} = [x_{pi}, 0]^T$ ,  $X_{wj} = [0, x_{wj}]^T$ ,  $X_{ck} = [x_{cpk}, x_{cwk}]^T$ ,  $D = [D_p, D_w]^T$

$$\begin{aligned} \min C_G(X_{pi}, X_{wj}, X_{ck}) = \\ \sum_{i=1}^{n_p} C_{pi}(X_{pi}) + \sum_{j=1}^{n_w} C_{ck}(X_{wj}) + \sum_{k=1}^{n_c} C_{wj}(X_{ck}) \end{aligned} \quad (4)$$

subject to the capacity, demand and process constraints in Equations 5, 6, and 7 respectively.

$$\begin{aligned} \underline{X}_{pi} \leq X_{pi} \leq \bar{X}_{pi} \quad \forall i = 1 \dots n_p \\ \underline{X}_{wj} \leq X_{wj} \leq \bar{X}_{wj} \quad \forall j = 1 \dots n_w \\ \underline{X}_{ck} \leq X_{ck} \leq \bar{X}_{ck} \quad \forall k = 1 \dots n_c \end{aligned} \quad (5)$$

$$\sum_{i=1}^{n_p} X_{pi} + \sum_{j=1}^{n_w} X_{wj} + \sum_{k=1}^{n_c} X_{ck} = D \quad (6)$$

$$\underline{r}_k \leq \frac{x_{cpk}}{x_{cwk}} \leq \bar{r}_k \quad \forall k = 1 \dots n_{cp} \quad (7)$$

where  $C_{pi}$ ,  $C_{wj}$ ,  $C_{ck}$  are the scalar cost functions for the  $i^{th}$  power production facility, the  $j^{th}$  water production facility and the  $k^{th}$  coproduction facility. Additionally,  $n_p$ ,  $n_w$ ,  $n_c$  are the numbers of power, water and coproduction facilities respectively.  $r_k^{upper}$  and  $r_k^{lower}$  are upper and lower bounds on the power-water production ratio for the coproduction plants. Here, the process constraints do not model the physical flows of power and water for coproduction facilities, as this would be intractable for all facilities. Instead, they represent the reasonable limits of safe operation of the coproduction process.  $D$  represents the power and water product demand vector. Finally, the

conventional overbar and underbar notation is used to reflect minimum and maximum capacity limits. Note that  $\underline{x}_{wj}$  and  $\underline{x}_{cwk}$  are allowed to take on negative values to account for the possibility of water storage in the system.

The cost functions  $C_{pi}$ ,  $C_{wj}$ ,  $C_{ck}$  are assumed to exhibit a quadratic structure in their respective production variables. This is a commonly held assumption for all three types of facilities: thermal power and coproduction facilities [56] and water treatment plants [145].

$$\begin{aligned}
 C_{pi} &= X_{pi}^T A_{pi} X_{pi} + B_{pi} X_{pi} + \mathcal{K}_{pi} \\
 C_{wj} &= X_{wj}^T A_{wj} X_{wj} + B_{wj} X_{wj} + \mathcal{K}_{wj} \\
 C_{ck} &= X_{ck}^T A_{ck} X_{ck} + B_{ck} X_{ck} + \mathcal{K}_{ck}
 \end{aligned} \tag{8}$$

The cost function coefficients are appropriately sized positive constant matrices based upon the heat rate characteristics of their respective production units.

The optimization model as presented may be classified as a nonlinear mathematical program subject to smooth monotonic constraints. With the exception of the process constraints, these constraints are linear. Therefore, this mathematical program is a practical candidate for existing off-the-shelf nonlinear optimization engines and has a high potential for industrial implementation within the time scale of 5 minutes to 1 hour.

#### IV. SIMULATION METHODOLOGY

Recalling Section 1.5, this real-time economic dispatch is the first in a three part series of optimization programs that have been recently developed [50]–[54]. In order to demonstrate consistency of approach between these contributions together, the common elements of the simulation methodology and data are used throughout. They are recounted here for clarity and convenience.

TABLE II  
PLANT AND COST DATA [51]–[53]

Plant Type	Index	Max Power Capacity (MW)	Max Water Capacity ( $m^3/hr$ )	Min Power Capacity (MW)	Min Water Capacity ( $m^3/hr$ )	Min Product Ratio ( $MWhr/m^3$ )	Max Product Ratio ( $MWhr/m^3$ )
Power	$i_1$	500	0	0	0	–	–
Power	$i_2$	400	0	0	0	–	–
Power	$i_3$	400	0	0	0	–	–
Power	$i_4$	350	0	0	0	–	–
Coproduction	$k_1$	800	200	160	30	4	9
Coproduction	$k_2$	600	150	120	23	4	9
Coproduction	$k_3$	400	100	80	15	4	9
Water	$j_1$	0	250	0	0	–	–

Power Plant Cost Coefficients			Water Plant Cost Coefficients		
$A_p$	$B_p$	$C_p$	$A_w$	$B_w$	$C_w$
2.069e-4	-1.483e-1	5.711e+1	1.816e-2	-7.081	7.374
3.232e-4	-1.854e-1	5.711e+1			
1.065e-3	-6.026e-1	1.268e+2			
4.222e-4	-2.119e-1	5.711e+1			

Coproduction Plant Cost Coefficients					
$A_{c11}$	$A_{c12}$	$A_{c22}$	$B_{c1}$	$B_{c2}$	$C_c$
4.433e-4	3.546e-3	7.093e-3	-1.106	-4.426	7.374e+2
7.881e-4	6.305e-3	1.261e-2	-1.475	-5.901	7.374e+2
1.773e-3	1.419e-2	2.837e-2	-2.213	-8.851	7.374e+2

The data used to demonstrate this optimization was selected for two reasons: 1.) The timing of power and water demand peaks and troughs is typical in the GCC. 2.) the range of the power and water demands is exaggerated to demonstrate the convergence capability of the selected optimization engine. The hypothetical test case is composed of 4 power plants, 3 co-production desalination facilities, and 1 reverse osmosis water plant. The associated plant

and cost data is summarized in Table II. Table III includes 24 hours of power and water demand data and a single optimization run is done for each of these hours.

Given the relatively well-behaved functional forms of the optimization program, it was sufficient to implement the optimization program with existing optimization engines for the numerical solution. The MATLAB and GAMS languages were used together; the former for data handling and visualization and the latter for optimization. The built-in CONOPT solver was selected for its efficient interior point algorithm for large scale nonlinear optimization problems [146].

TABLE III  
POWER & WATER DEMAND DATA [51]–[53]

Hour	Power Demand (MW)	Water Demand ( $m^3/hr$ )
1	1250	150
2	1125	130
3	875	100
4	750	150
5	950	200
6	1440	350
7	1500	300
8	1750	200
9	2000	300
10	2250	400
11	2500	500
12	2750	600
13	2875	400
14	3250	400
15	2750	500
16	2500	550
17	2125	550
18	2375	500
19	2250	400
20	1975	350
21	1750	300
22	1625	250
23	1500	200
24	1376	150

## V. RESULTS

The results of the optimization include: the dispatch levels of power and water in Figures 1 and 2 respectively, the power to water ratio for the coproduction plants relative to the power and water demand ratio in Figure 3, and the total costs incurred over the 24 hour period in Figure 4. Each of these figures is discussed in turn.

Figures 1 and 2 show the power and water generation profiles over the 24 hours respectively. The optimization successfully completed in spite of a more than a 4x variation in power demand over the course of the day. These exaggerated peaks and troughs for both power and water represent more demanding optimization conditions than those commonly found in power demand profiles in real life dispatch. The power and water demand profiles are also not necessarily trending together leading to a significant variation in the demanded power to water ratio over the course of the day. These demand profiles were chosen in a manner so as to reflect the common power and water demand profiles observed in real life dispatch. In power demand, the peak is typically in the afternoon, when maximum power is utilized by industrial areas, offices etc. The lowest levels of power required are typically early in the morning and later on in the evening. Water demand has an early peak for irrigation and domestic use and another peak around midday for industrial & commercial use.

Interestingly, the co-production facilities act as units of ‘first-choice’. The single product power and water plants are essentially being used as peaking plants; coming into operation only to meet periods of high demand of their

respective products. This occurs for two reasons. First, the heat rate data for the coproduction plants, relatively speaking, has a much more exaggerated downward trend making them more economical to run close to capacity whenever possible. The second becomes apparent from Figure 3.

Figure 3 elucidates one cause of the coproduction facilities being used as units of ‘first-choice’. As the power to water demand ratio swings over the course of the day, the cogeneration facilities are not just incentivized to run as close to full capacity as possible but also are constrained by their product ratio process constraint. Internal to a coproduction facility, there exist mass and energy balance laws that fundamentally limit the range of the power to water ratio. In this simulation, these limits of safe operation were set to the typical values of  $4 \leq R \leq 9 \text{ MW}/\text{m}^3$ . The figure shows that the demand ratio swings significantly causing each of the coproduction facilities to track accordingly. As a result, the single product power and water plants are essentially crowded out; coming online as units of last resort during peak demand hours. The tracking behavior of the coproduction plants ultimately suggests that any process flexibility that can be achieved by dual product desalination units could lead to significantly improved optima.

Finally, Figure 4 shows the total costs of generation. At first glance, the results seem counter-intuitive with higher total costs during periods of low production. Once again, this arises from the fact that the coproduction heat rates are higher than single product plants in absolute terms for all production levels and also exhibit a much sharper downward trend for all production levels. As a result, costs are dominated by the coproduction facilities which were only dispatched due to their process constraints. The high cost of low demand arises from the fact that any incremental decreases in demand are more than compensated by increases in the corresponding heat rate.

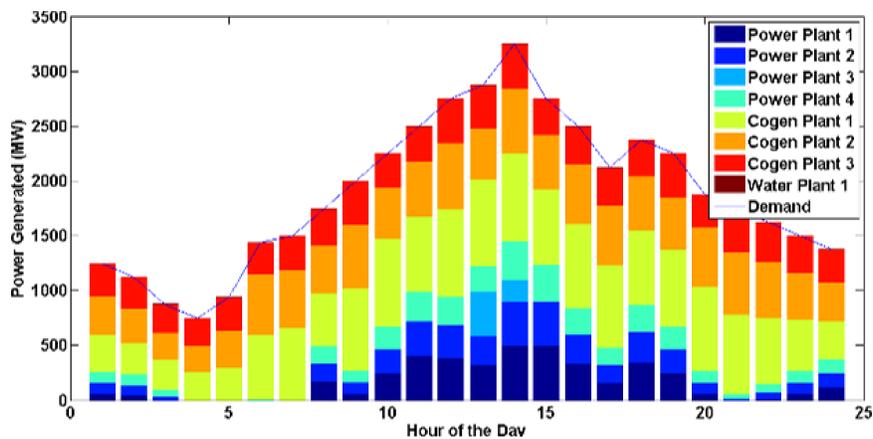


Fig. 1. Power generation and demand profile over 24 hour period

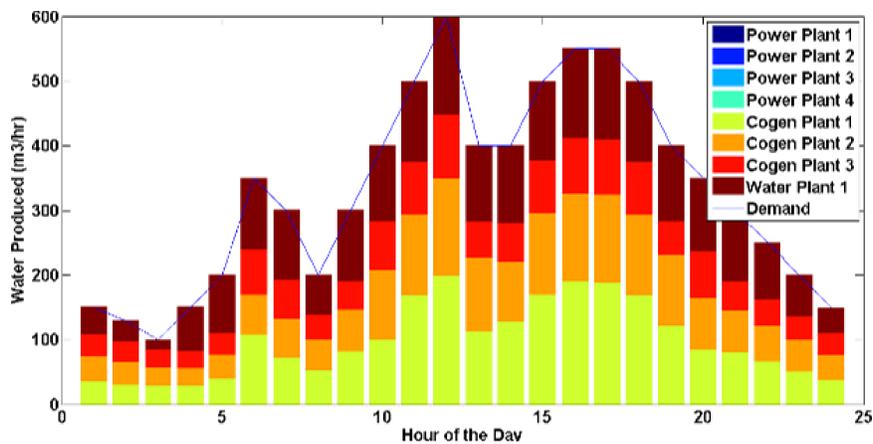


Fig. 2. Water production and demand profile over 24 hour period

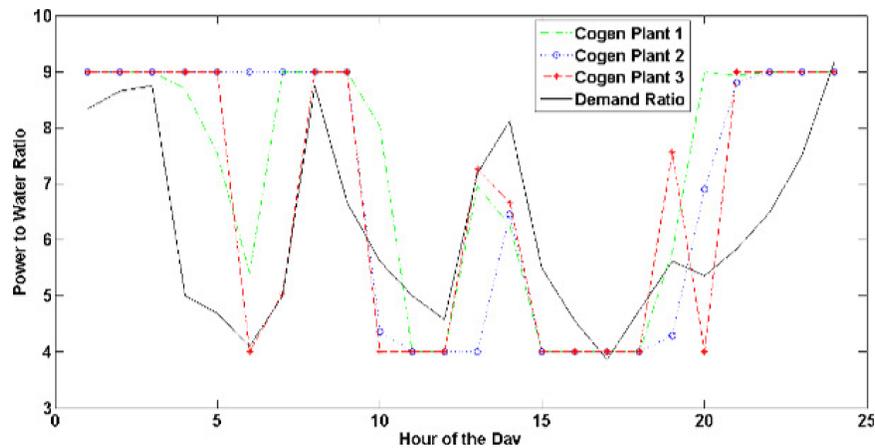


Fig. 3. Power to water ratio for the cogenerator plants

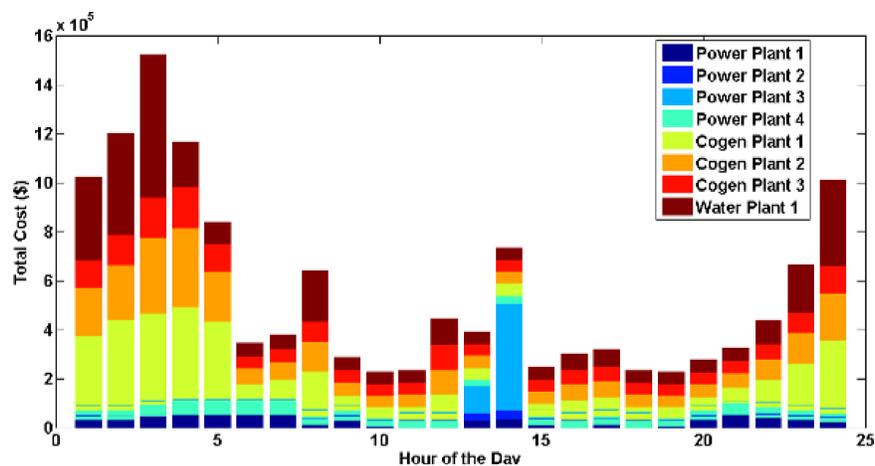


Fig. 4. Cost incurred by different units over the period of 24 hours

## VI. CONCLUSIONS AND FUTURE WORK

The co-optimization program distinguishes itself from previous work in that it allows for deciding set points on the basis of which individual plants can plan their optimal operation. Total costs as a function of power and water generation subject to the demand, capacity and process constraints were minimized on a hypothetical system composed of four power plants, three production facilities and one water plant. Interesting results were observed suggesting that the coproduction minimum capacity limits and process constraints can lead to scenarios where cheaper single product plants can be crowded out of the dispatch. Such results suggest that water and/or power storage can have an important role in relieving process constraints and reducing costs [52], [53].

The economic dispatch seeks to optimize the use of resources both in terms of costs and efficiencies. By systematically employing an economic dispatch, there is a trend to dispatch plants that are more energy efficient. Thus, plants are incentivized to improve their efficiencies by using better fuels and better technologies. This is especially relevant for the simultaneous co-optimization of power and water since it encourages more efficient use of fuels by producing dual products while at the same time providing two vital resources in an economical manner. That said, the co-optimization does present challenges that would not be found in either of the single product cases alone.

As has been observed, there has been a strong coupling between water and power and this is more likely to increase in coming years as demand for both increase. There are two ways to handle this. One possible option is to try to reduce coupling between the two products. Water plants based upon reverse osmosis technology do require significant electrical input but they avoid coupling power generation with water production as in MSF plants. However, this generally is only applicable to new plant installation and much less so to retrofitting scenarios. The

other option is to better understand the coupling between these two resources and use a well established algorithm to optimize their production. For example, much attention has recently been given to the integration of renewable energy into desalination facilities [29].

Operations research in the energy-water nexus is an area where there is a lot of potential for future development. As traditional power economic dispatch has evolved over the last few decades, the combined optimization of power and water can evolve similarly. Organic developments that incorporate physical constraints like ramping, transmission losses and start-ups are likely to appear. There is also potential to examine how to better optimize dispatch, by including, for example, power and water storage. Given the ease of water storage and the growing potential for power storage, initial work has been done on the impact of storage facilities on the supply side of energy-water nexus [52], [53]. Similarly, initial developments on the incorporation of transmission constraints are reported [51].

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