IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received September 12, 2021, accepted September 25, 2021, date of publication October 5, 2021, date of current version October 14, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3118010

A Dynamic System Model for Personalized Healthcare Delivery and Managed Individual Health Outcomes

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ABSTRACT The current healthcare system is facing an unprecedented chronic disease burden. This paper develops a dynamic mathematical simulation model for personalized healthcare delivery and managed individual health outcomes. It utilizes a highly novel hetero-functional graph theory, most easily understood as an ontological convergence of model-based systems engineering and network science, along with Petri nets. The dynamics of the model builds upon a recently developed systems architecture for healthcare delivery, which bears several analogies to the architecture of mass-customized production systems. At its essence, the model consists of two synchronized Petri nets; one for the healthcare delivery system and another for individuals' health state evolution. The dynamic model links logistical and medical phenomena using a combination of a deterministic Petri net model for the former and a fuzzy Petri net for the latter. The model is demonstrated on two clinical case studies; acute and chronic. Together, the case studies show that the model applies equally to the care of both acute and chronic conditions, transparently describes health outcomes and links them to the evolution of the healthcare delivery system and its associated costs.

INDEX TERMS Systems framework, dynamic model, healthcare delivery system, personalized care, individual health outcomes, discrete-event modeling, Petri net.

I. INTRODUCTION

Growing healthcare costs have drawn significant attention to the healthcare delivery system and its fragile and fragmented nature. Similarly, the growing burden of illness has also directed attention to addressing patients' health needs. The consequences of the growing burden of illness compounded by an increasingly expensive healthcare delivery system places grave consequences on our economy and way of life.

Efforts to affect positive change requires an understanding of the complex dynamics of healthcare delivery systems and patients' health. Most modeling focuses on either 1.) the *healthcare delivery system* that renders patients without state as they are pushed and pulled through the system (e.g., a patient with an acute condition in an ER) or 2.) the *patient health* without any consideration of its interface with the healthcare delivery system. In order to develop a dynamic system model of a personalized healthcare delivery system in which individual health outcomes are managed, these two processes need to be linked.

A. DYNAMIC MODELING OF HEALTHCARE DELIVERY SYSTEMS

Efforts to quantitatively and dynamically model the healthcare delivery system originate from the field of production systems [1]. Production system modeling focuses on transporting operand-products one from location to another in the system. The analogous extension to healthcare delivery system as a type of production system treats the patient as an operand-product as well. In doing so, operand throughput and system efficiency in terms of cost and time can be quantitatively assessed and maximized.

Consequently, a large body of healthcare literature summarizes safety [2], [3], resource management [4], capacity planning [5], [6], and scheduling of various types including for outpatients [7]–[10], clinicians [11]–[13], operating room [14]–[16], work flow processes [17]–[20], and patient-flows [21].

The associate editor coordinating the review of this manuscript and approving it for publication was Zhiwu Li.

Healthcare is well-suited to apply production modeling methods for patients with acute conditions. The focus is on addressing the urgency of the patient's health state by quickly transporting the patient through the healthcare delivery system before the patient falls into more serious diagnoses. Such modeling assumes that the operand's state can be described by its position in the system in much the same way a product's state can be inferred based upon its relative position in a production system.

Chronic conditions, however, have a much longer *time-scale* and require a different approach. Moving patients through the healthcare delivery system efficiently (i.e., faster) does not address disease understanding or affect long-term disease course. For example, addressing pain for a patient with a chronic condition (e.g., rheumatoid arthritis, cancer), may prevent recurring emergency department visits all-together and consequently modeling transportation during these visits is entirely superfluous. Processing the acute visit and moving the patient through the ER faster, however, does not address the need for long-term pain care. Understanding health state and factors affecting health status, be they physical or social determinants of health (e.g., socioeconomic status, physical environment) [22], is critical when understanding an individual's interaction with the healthcare delivery system. Given that chronic conditions now account for 78% of healthcare delivery system expenditures [23], it is most important that modeling efforts focus on the most essential features of chronic care.

B. DYNAMIC MODELING OF HEALTH

Health has been modeled at many levels of granularity; from the cell level to the disease level. The field of systems biology focuses on continuous-time modeling at a cellular scale of bio-physical-chemical processes [24], [25]. In contrast, clinical medicine generally focuses on discrete modeling of diseases (e.g., diabetes [26], cancer [27]).

C. PAPER CONTRIBUTION

This paper develops the dynamics for a system model for personalized healthcare delivery and managed individual health outcomes. This paper builds upon previous works [28], with extensions to the model and inclusion of cases and numerical validation of the model for acute and chronic condition. The goal of this dynamic mathematical simulation model is to quantitatively characterize, understand, and validate the dynamic behavior of healthcare systems. Unlike classic healthcare operations management simulation, the goal here is to develop a simulation model that elucidates dynamic system behavior rather than optimizes an objective with a set of constraints. The model is validated in two examples (both acute and chronic) drawing on the data from book case sources. It utilizes hetero-functional graph theory [29], most easily understood as an ontological convergence of model-based systems engineering and network science, along with Petri nets. The dynamics of the model builds upon the developed systems architecture for healthcare delivery in [30], which bears several analogies to the architecture of mass-customized production systems. At its essence, the model consists of two synchronized Petri nets; one for the healthcare delivery system and another for individuals' health state evolution. This synchronization is a novel approach to coordinate the health state of individuals with healthcare delivery systems. The model applies equally to the care of both acute and chronic conditions, transparently describes health outcomes and links them to the evolution of the healthcare delivery system and its associated costs. The presence of these two Petri nets allows us to define a heuristic method that differentiates between the dynamics of acute and chronic healthcare delivery. The subject of making optimized control and operations management decisions is left to future work once this dynamic system model is established.

These contributions are novel in three ways. First, this is the first application of hetero-functional graph theory to develop a dynamic mathematical simulation model for personalized healthcare and managed individual health outcomes. Hetero-functional graph theory, as a generalization, can address systems of arbitrary heterogeneity and where transportation activities are not an important part of the problem (as in the case of chronic care). The ability to address a wide range of systems leads to the second novelty, in which the developed system model applies equally to acute and chronic care. Finally, the synchronization of the two Petri nets, for the first time, links logistical and medical phenomena to coordinate the health state of individuals with healthcare delivery systems.

D. PAPER OUTLINE

The model's development rests upon a previously established architectural foundation [30]. Section II presents the essential definitions and concepts from this foundation so that Section III may develop a conceptually consistent dynamic model. Two illustrative examples are presented to demonstrate applicability to both acute and chronic care. The acute care example begins in Section II and serves as a concrete example of the abstract hetero-functional concepts. The chronic care example is presented in Section IV. The discussion is presented in Section V and, finally, the conclusion in Section VI. The work assumes prerequisite knowledge in model-based systems engineering [31]–[34], graph theory [35], [36], and discrete-event simulation [37] which is otherwise gained from the cited texts.

II. BACKGROUND: PRELIMINARIES

The development of the dynamic model in Section III rests upon the recently developed architecture for personalized healthcare delivery and managed individual health outcomes [30]. That work drew upon a hetero-functional graph theory [29], an ontological convergence of model-based systems engineering and network science, and utilizes Petri nets. The healthcare delivery system form is described by its resources in Section II-B, and its system function is described by processes in Section II-C. The processes are allocated to resources in the system concept as described by the system knowledge base in Section II-D.

The purpose of this section is to provide an exposition of the fundamental concepts of hetero-functional graph theory to an uninitiated healthcare audience. In order to facilitate this goal, an acute care illustrative example is described below. This example will then serve as a basis for providing a concrete explanation to the relatively abstract concepts of hetero-functional graph theory.

A. DESCRIPTION OF ORTHOPEDIC CASE

An example orthopedic case study of an ACL injury & repair is described below; drawing from a textbook clinical case [38].

Case Study 1: 'Adam injured his left knee playing rugby when he fell forwards and sideways while the left foot remained fixed on the ground. He felt immediate pain and was unable to continue with the game. Pain and swelling increased over the next 2 *hours. He was seen in an emergency department and X-rays were negative for fractures. He was prescribed anti-inflammatories, given elbow crutches and advice on ice, rest and elevation. A clinic appointment to see an orthopedic consultant was arranged.*

The orthopedic clinician evaluated the individual through a battery of special tests: anterior drawer test and valgus stress instability and active Lachman's test all of which were not conclusive due to pain and swelling. The individual received an urgent MRI scan which showed a rupture of the left ACL and a medial collateral ligament tear. Surgery was performed followed by an ACL post-operative rehabilitation protocol at physical therapy'. [38]

B. SYSTEM FORM

The healthcare delivery system is composed of resources representing system form. Four types of resources $\mathbb{R} = \mathbb{R}_F \cup \mathbb{R}_D \cup \mathbb{R}_M \cup \mathbb{R}_N$ have been defined [30]. A **transformation resource** $\mathbb{r}_F \in \mathbb{R}_F$ is capable of a transformative effect on its operand (e.g., the health state of an individual). A **decision resource** $\mathbb{r}_D \in \mathbb{R}_D$ is capable of advising the operand, an individual, on how to proceed next with the healthcare delivery system. A **measurement resource** $\mathbb{r}_M \in \mathbb{R}_M$ is capable of measuring the operand–here the health state of an individual. A **transportation resource** $\mathbb{r}_N \in \mathbb{R}_N$ is capable of transporting its operand–the individual themself. For each of the four types of resources, each resource, $\mathbb{R}$, is the set union of the human resources, R, and technical resources, $\mathscr{R}$. For example, $\mathbb{R}_F = R_F \cup \mathscr{R}_F$. In the cases where a specific resource is capable of performing several processes, it must be uniquely classified. For human resources, if $r \in R$ can *Transform*; then $r \in R_F$, then if $r \in R$ can *Decide*; then $r \in R_D$, then if $r \in R$ can *Measure*; then $r \in R_M$, otherwise $r \in R_N$. The same applies to technical resources.

We interpreted the orthopedic case study 1 text to identify the system resources. We modeled each sentence to identify system form. While the narrative describes several levels of detail, we modeled at a high-level to keep the example simple. For example, the sentence "He was seen in an emergency department and X-rays were negative for fractures" describes two resources: emergency department and radiology department. Overall, we modeled five system resources (emergency department, orthopedic department, physical therapy department, radiology department, and transportation staff&equipment). Each resource was classified into the highest-level process it could perform. Additionally, an 'outside clinic' resource is added to allow for entry into and out-of the healthcare system to reflect the case's three clinical visits. We modeled 'outside clinic' as a transformational resource since it is a place where transformation can occur (e.g., knee injury). An additional transportation resource called 'vehicle' is introduced to transport individuals from 'outside clinic' to the specific department resources. Altogether, we modeled a mutually exclusive and collectively exhaustive set of seven system resources: 4 transformational resources: $\mathbb{R}_F$ = {outside clinic, emergency department, orthopedic department, physical therapy department}; one decision resource, $\mathbb{R}_D$ = {radiology department}; and two transportation resources, $\mathbb{R}_N$ = {transportation staff&equipment, vehicle}.

C. SYSTEM FUNCTION

The healthcare delivery system is composed of processes $P = P_F \cup P_D \cup P_M \cup P_N$ representing system function. Four types of processes have been previously defined [30]. A **transformation process** is a *physical* process $p_F \in P_F$ that transforms the operand–specifically the internal health state of the individual (i.e., treatment of condition, disease or disorder). A **decision process** is a *cyber-physical* process $p_D \in P_D$ occurring between a healthcare system resource and the operand: the individual, that generates a decision on how to proceed next with the healthcare delivery system. A **measurement process** is a *cyber-physical* process $p_M \in P_M$ that converts a physical property of the operand into a cyber, informatic property to ascertain health state of the individual. A **transportation process** is a *physical* process $p_N \in P_N$ that moves individuals between healthcare resources (e.g., bring individual to emergency department, move individual from operating to recovery room).

We identified a mutually exclusive and collectively exhaustive set of system processes for orthopedic case study 1. We model both activities explicitly detailed in the narrative and processes that must occur to reflect the narrative. For example, the sentence, "He was seen in an emergency department and X-rays were negative for fractures" suggests that Adam was 1) checked into the emergency department as a patient, 2) transported to the radiology department for an X-ray scan, and 3) a decision was made that the X-ray was negative for fractures. Again, we classify each process as transformation, decision, measurement, or transportation. We identified a total of thirty-three processes: four transformational processes, P_F = {Perform check in, Perform check out, Perform surgical procedures, Perform therapeutic procedures}; two decision processes, P_D = {Decide on care planning, Decide on care scheduling}; two measurement processes, P_M = {Perform physical exam evaluation, Perform diagnostic testing}; and twenty-five possible transportation processes between the five non-transportation resources.

D. SYSTEM CONCEPT

The system concept is defined as an allocated architecture composed of a bipartite graph between system processes and resources, that can be mathematically described as [39]–[46], $P = J_S \odot \mathbb{R}$, where J_S is the system knowledge base and $\odot$ is boolean multiplication. The **system knowledge base** J_S of size $\sigma(P) \times \sigma(\mathbb{R})$ whose element $J_S(w, v) \in \{0, 1\}$ is equal to one when event $e_{wv} \in \mathcal{E}_S$ (in the discrete event systems sense [37]) exists as a system process $p_w \in P$ being executed by a resource $r_v \in \mathbb{R}$. The healthcare delivery system knowledge base J_S represents the elemental capabilities that *exist* within the system. These capabilities may not always be *available* and therefore such constraints can be described in a similar structure called the system events constraints matrix. The **system events constraints matrix** K_S is a binary matrix of size $\sigma(P) \times \sigma(\mathbb{R})$ whose element $K_S(w, v) \in \{0, 1\}$ is equal to one when a constraint eliminates event e_{wv} from the event set. The construction of J_S and K_S allow for the construction of a *system concept matrix* A_S describing the independent actions defining the available capabilities in the system [39]–[46], $A_S = J_S \ominus K_S$, where $\ominus$ is Boolean subtraction. The enumeration of these independent actions defines the healthcare system's structural degrees of freedom.

Definition 1 (Structural Degrees of Freedom [39]–[46]): The set of independent actions $\psi_i \in \mathcal{E}_S$ that completely defines the available processes in the system. It is given by:

$$DOF_S = \sigma(\mathcal{E}_S) = \sum_{w}^{\sigma(P)} \sum_{v}^{\sigma(\mathbb{R})} A_S(w, v) \quad (1)$$

From an architectural perspective, the structural degrees of freedom form the nodes of a hetero-functional network [42], [47] that describes the structure of the healthcare delivery system. It is often useful to vectorize the knowledge base, where the shorthand $()^V$ is used to replace vec(). A projection operator may be introduced to project the vectorized knowledge base onto a one's vector to eliminate sparsity. $\mathbb{P}(A_S)^V = \mathbb{1}^{\sigma(\mathcal{E}_S)}$

$$\mathbb{P} = \left[e_{\psi_1}^{\sigma(\mathcal{E}_S)}, \ldots, e_{\psi_{\sigma(\mathcal{E}_S)}}^{\sigma(\mathcal{E}_S)} \right] \quad (2)$$

where $e_{\psi_1}^{\sigma(\mathcal{E}_S)}$ is the ψ_i^{th} elementary row vector corresponding to the first up to the last structural degree of freedom. In summary, the variables in the healthcare delivery structural system model are summarized in Table 1.

We construct the system knowledge base J_S as shown in Figure 1 using the identified resources and processes. A black box indicates a value of one where the process in the row position can be executed by the resource in the column position. The text does not indicate any event constraints. $K_S = 0$. Consequently, $J_S = A_S$. We calculated the structural degrees of freedom using Equation 1. The enumerated number in the black box indicates the index for each degree of freedom. $DOF_S = 42$.

TABLE 1. Healthcare Delivery Structural System Model Variables.

SYSTEM	PERSONALIZED HEALTHCARE DELIVERY SYSTEM
(A) System Form	
Resources	transformation($\mathbb{R}_F$) ∪ decision($\mathbb{R}_D$) ∪ measurement($\mathbb{R}_M$) ∪ transportation($\mathbb{R}_N$)
Resource Classification	transform>decide>measure>transportation
(B) System Function	
Processes	transformation(P_F) ∪ decision(P_D) ∪ measurement(P_M) ∪ transportation(P_N)
(C) System Context	
System Knowledge Base	$J_S = \begin{bmatrix} J_F & 0 & 0 & 0 \\ J_{FD} & J_D & 0 & 0 \\ J_{FM} & J_{DM} & J_M & 0 \\ J_{FN} & J_{DN} & J_{MN} & J_N \end{bmatrix}$
System Constraint Matrix	$K_S = \begin{bmatrix} K_F & 0 & 0 & 0 \\ K_{FD} & K_D & 0 & 0 \\ K_{FM} & K_{DM} & K_M & 0 \\ K_{FN} & K_{DN} & K_{MN} & K_N \end{bmatrix}$
System Availability Matrix	$A_S = J_S \ominus K_S$
Structural Degrees of Freedom	$DOF_S = \sigma(\mathcal{E}_S) = \sum_{w}^{\sigma(P)} \sum_{v}^{\sigma(\mathbb{R})} A_S(w, v)$

Resources / Processes

			R_F: R_{F1} outside clinic	R_F: R_{F2} emergency dept.	R_F: R_{F3} orthopedic dept.	R_F: R_{F4} physical therapy dept.	R_D: R_D radiology dept.	R_N: R_{N1} transportation staff & equip	R_N: R_{N2} vehicle
P_F	P_{F1}	Perform check in		2	10	18			
	P_{F2}	Perform check out		3	11	19			
	P_{F3}	Perform surgical procedure		4	12				
	P_{F4}	Perform therapeutic procedure		5	13	20			
P_D	P_{D1}	Decide on care planning		6	14	21	25		
	P_{D2}	Decide on care scheduling		7	15	22	26		
P_M	P_{M1}	Perform physical exam evaluation		8	16	23			
	P_{M2}	Perform diagnostic testing					27		
P_N	P_{N1}	Transport from F1 to F1	1						
	P_{N2}	Transport from F1 to F2							35
	P_{N3}	Transport from F1 to F3							36
	P_{N4}	Transport from F1 to F4							37
	P_{N5}	Transport from F1 to D							38
	P_{N6}	Transport from F2 to F1							39
	P_{N7}	Transport from F2 to F2		9					
	P_{N8}	Transport from F2 to F3							
	P_{N9}	Transport from F2 to F4							
	P_{N10}	Transport from F2 to D						29	
	P_{N11}	Transport from F3 to F1							40
	P_{N12}	Transport from F3 to F2							
	P_{N13}	Transport from F3 to F3			17				
	P_{N14}	Transport from F3 to F4							
	P_{N15}	Transport from F3 to D						30	
	P_{N16}	Transport from F4 to F1							41
	P_{N17}	Transport from F4 to F2							
	P_{N18}	Transport from F4 to F3							
	P_{N19}	Transport from F4 to F4				24			
	P_{N20}	Transport from F4 to D						31	
	P_{N21}	Transport from D to F1							42
	P_{N22}	Transport from D to F2						32	
	P_{N23}	Transport from D to F3						33	
	P_{N24}	Transport from D to F4						34	
	P_{N25}	Transport from D to D					28		

FIGURE 1. Acute Care Healthcare Delivery System Knowledge Base J_S. Filled elements represent allocated processes to resources. Since we identified no constraints (i.e., $K_S = 0$, J_S is also equal to A_S. Thus, the enumerated number in the black box indicates the index for each degree of freedom.

III. DYNAMIC MODEL DEVELOPMENT

The structural model presented in the previous section provides a skeleton upon which to develop the dynamic model in this section. Because healthcare delivery systems are spatially distributed and evolve with discrete-event dynamics, the dynamic model utilizes Petri nets [37]. Two types are needed. The first is called the *Healthcare Delivery System Petri Net*. It describes the evolution of the system processes and resources of the healthcare delivery system in Section III-A. Section III-B then refines this default model to the care of chronic conditions. The second Petri net is called the *Health Net*. It describes the 'clinical' health state evolution of individuals in Section III-C. As discussed in detail previously [48], although the human body's health state evolves continuously via biological processes, the practice of clinical medicine discretizes this evolution into discrete states so as to facilitate diagnosis and decision-making. With these two Petri nets in place, their respective dynamics are synchronized in Section III-D.

A. HEALTHCARE DELIVERY SYSTEM DYNAMICS

The healthcare delivery system dynamics are described by a timed Petri net.

Definition 2 (Healthcare Delivery System Petri Net): A bipartite directed graph represented as a 6-tuple:

$$N = \{S, \mathcal{E}, \mathcal{M}, W, D, Q\} \tag{3}$$

where:

- N is the Healthcare Delivery System net.
- S is the set of places (or buffers) of size $\sigma(\mathbb{R}_B)$.
- $\mathcal{E}$ is the set of transitions/events of size $\sigma(\mathcal{E}_S)$, also equal to DOF_S from Equation 1.
- $\mathcal{M} \subseteq (S \times \mathcal{E}) \cup (\mathcal{E} \times S)$ is the set of arcs of size $\sigma(\mathcal{M})$ from places to transitions and from transitions to places.
- $W : \mathcal{M} \rightarrow \{0, 1\}$ is the weighting function on arcs.
- D is the set of transition durations.
- Q is a discrete state marking vector of size $\big(\sigma(\mathbb{R}_B) + \sigma(\mathcal{E}_S)\big) \times 1 \in \mathbb{N}^{\sigma(\mathbb{R}_B)+\sigma(\mathcal{E}_S)}$.

In the model, there is exactly one *place* for each healthcare system buffer. As many healthcare systems may have hundreds or thousands of healthcare system buffers, it is often useful to form aggregated resources $\bar{\mathbb{R}}$ [39]–[41], [44], [49].

$$\bar{\mathbb{R}} = A_R \circledast \mathbb{R} \tag{4}$$

where $\circledast$ is an aggregation operator and A_R is an aggregation matrix [39]–[41], [44], [49] and $A_R(i, j) = 1$ iff $\mathbb{R}_j \in \bar{\mathbb{R}}_i$. For example, a human resource such as a surgeon must be aggregated with a technical resource such as an operating room in order to make a functional surgical theatre.

In the model, there is exactly one *transition* for every structural degree of freedom in the system. This allows for all the capabilities of the healthcare delivery system to be potentially engaged by the patient population. It is also important to note that the healthcare delivery system knowledge base can show process redundancies where a given process can be performed by multiple resources. This critical distinction allows two different transitions to be fired and achieve the same process but engage entirely different resources at entirely different cost. For example, a process 'perform skin suturing' performed by a resource 'resident' vs. 'plastic surgeon' have different costs associated with each transition.

The (directed) *arcs* of the Petri net graph and their weightings define the Petri net incidence matrix $\mathcal{M}$.

Definition 3 (Petri Net Incidence Matrix [50]): An incidence matrix $\mathcal{M}$ of size $\sigma(\mathbb{R}_B) \times \sigma(\mathcal{E}_S)$ where:

$$\mathcal{M} = \mathcal{M}^+ - \mathcal{M}^- \tag{5}$$

where $\mathcal{M}^+(y, \psi) = \mathrm{w}(\epsilon_{wv}, r_y)$ and $\mathcal{M}^-(y, \psi) = \mathrm{w}(r_y, \epsilon_{wv})$ and ψ is a unique index mapped from the ordered pair (w, v).

The incidence out and incidence in matrices ($\mathcal{M}^-$ and $\mathcal{M}^+$) form the negative and positive components of the Petri net incidence matrix respectively. The incidence out matrix may be calculated straightforwardly [45]. For a more detailed mathematical explanation, the reader is referred to [45].

$$\mathcal{M}^- = \sum_{y1=1}^{\sigma(\mathbb{R}_B)} e_{y1}^{\sigma(\mathbb{R}_B)} \left[\mathbb{P}\left(X_{y_1}^-\right)^V\right]^T \tag{6}$$

where,

$$X_{y_1}^- = \left[\frac{\mathbb{1}^{\sigma(P_B)} e_{y1}^{\sigma(\mathbb{R}_B)T} \quad | \quad \mathbf{0}^{\sigma(P_B)\times\sigma(\mathbb{R}_N)}}{e_{y1}^{\sigma(\mathbb{R}_B)} \quad \otimes \mathbb{1}^{\sigma(\mathbb{R}_B)} \quad \otimes \mathbb{1}^{\sigma(\mathbb{R})T}}\right] \tag{7}$$

Equation 6 states that the incidence matrix is the linear superposition of $\sigma(\mathbb{R}_B)$ matrices each associated with a given Petri net buffer r_{y1}. For a given buffer r_{y1}, the outer product serves to link it to its associated structural degree of freedom or equivalently a Petri net transition. Note that the matrix $X_{y_1}^-$ has the same size and structure as the system knowledge base J_S and when projected by $\mathbb{P}$ (in Equation 2) serves to select out the elements aligned with the structural degrees of freedom. Finally, the $X_{y_1}^-$ matrix defined in Equation 7 simply places filled elements at the structural degrees of freedom that 1.) occur at r_{y1} and 2.) have r_{y1} as its origin. The incidence in matrix may be calculated analogously [45].

$$\mathcal{M}^+ = \sum_{y2=1}^{\sigma(\mathbb{R}_B)} e_{y2}^{\sigma(\mathbb{R}_B)} \left[\mathbb{P}\left(X_{y_2}^+\right)^V\right]^T \tag{8}$$

where,

$$X_{y_2}^+ = \left[\frac{\mathbb{1}^{\sigma(P_B)} e_{y2}^{\sigma(\mathbb{R}_B)T} \quad | \quad \mathbf{0}^{\sigma(P_B)\times\sigma(\mathbb{R}_N)}}{\mathbb{1}^{\sigma(\mathbb{R}_B)} \quad \otimes e_{y2}^{\sigma(\mathbb{R}_B)} \quad \otimes \mathbb{1}^{\sigma(\mathbb{R})T}}\right] \tag{9}$$

Equation 8 is constructed similarly to Equation 6, but now the calculation is performed for a given Petri net buffer r_{y2} to distinguish between incoming and outgoing Petri net buffer. Similar to Equation 7, Equation 9 simply places filled elements at the structural degrees of freedom that 1.) occur at r_{y2} and 2.) have r_{y2} as its destination. For a more detailed mathematical explanation, the reader is referred to [45].

The healthcare delivery system Petri net model for the orthopedic case is constructed as shown in Figure 2. A single Petri net place is shown for each of the five non-transportation resources. A single transition is shown for each of the forty-two structural degrees of freedom. The places and transitions have been graphically situated to reflect their 'physical' location in the healthcare clinic. The arcs with a value of 1 are visualized connecting places to transitions and transitions to places. It is worth noting that the healthcare delivery system Petri net model reflects *possible* physical places outside and within the clinic, and possible system capabilities as transitions, to reflect real-world healthcare system capabilities. The actual transitions fired depend on the specific case. The initial discrete marking vector at the beginning includes one marking in the place labeled 1 'outside the clinic', corresponding to the individual Adam outside the healthcare clinic prior to his injury and his need to enter the healthcare delivery system. Since many of the degrees of freedom are non-transportation, many transitions start and return to the same place. However, there is minimal concern for deadlocks and livelocks, because the Petri net does not have the situation where there is mathematically any difficulty exiting a place after entry.

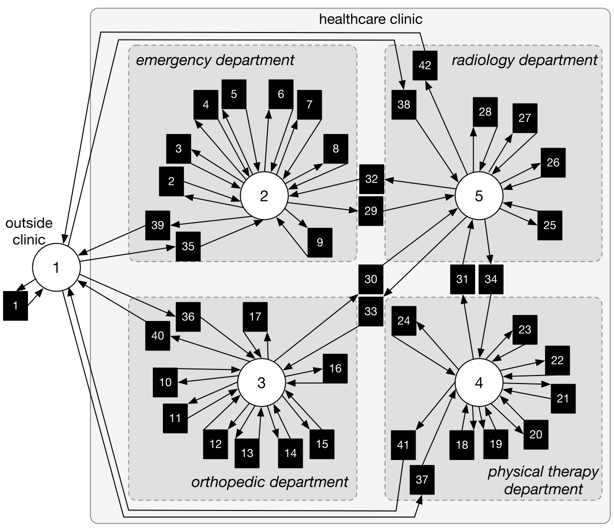

FIGURE 2. Acute Care Healthcare Delivery System Petri net. The places (*circles*), represent buffer resources={outside clinic, reception, emergency room, imaging, physical therapy, orthopedic surgery}. The transitions (*rectangles*), are numbered based on the healthcare delivery system structural degrees of freedom index ψ_i.

The Petri net structure leads directly to the definition of its timed discrete-event dynamics.

Definition 4 (Timed Petri Net (Discrete-Event) Dynamics [39]–[46]): Given a binary input firing vector $U^+[k]$ and a binary output firing vector $U^-[k]$ of size both of size $\sigma(\mathcal{E}_S)\times 1$, and the positive and negative components $\mathcal{M}^+$ and $\mathcal{M}^-$ of the Petri net incidence matrix of size $\sigma(\mathbb{R}_B)\times\sigma(\mathcal{E}_S)$, the evolution of the marking vector Q is given by the state transition function $\Phi_T(Q[k], U[k])$:

$$Q[k+1] = \Phi_T(Q[k], U^-[k], U^+[k]) \tag{10}$$

where $Q = [Q_S; Q_{\mathcal{E}_S}]$ and

$$Q_S[k+1] = Q_S[k] + \mathcal{M}^+U^+[k] - \mathcal{M}^-U^-[k] \tag{11}$$
$$Q_{\mathcal{E}_S}[k+1] = Q_{\mathcal{E}_S}[k] - U^+[k] + U^-[k] \tag{12}$$

The state transition function breaks the discrete state Q in two. Q_S tracks the locations of the tokens at the places $\mathbb{R}_B$ as a vector of size $1\times\sigma(\mathbb{R}_B)$, that evolves at each new k+1 step, following Equation 11. $Q_{\mathcal{E}_S}$ tracks the locations of the tokens in the transitions $\mathcal{E}_S$ of the healthcare delivery system as a vector of size $1\times\sigma(\mathcal{E}_S)$, that evolves at each new k+1 step, following Equation 12. The state transition function also distinguishes between the input firing vector, $U^+[k]$, and the output firing vector, $U^-[k]$, so as to mark the entry and exit of tokens to and from transitions. In practice, a scheduled event list is used to implement firing vectors and ensure the durations D of each of the transitions.

Definition 5 (Scheduled Event List [37]): A tuple $\mathcal{S} = (u_\psi[k], t_k)$ consisting of all elements $u_\psi[k]$ in firing vectors $U^-[k]$ and their associated times t_k. For every element, $u^-_\psi[k]\in U^-[k]$, there exists another element $u^+_\psi[\kappa]\in U^+[\kappa]$ which occurs at time t_κ, d_ψ time units later. $t_\kappa = t_k + d_\psi$.

The orthopedic case study 1 narrative is rewritten as a string of healthcare delivery system events $\mathcal{E}_S$ as shown in Figure 3. Each event in $\mathcal{E}_S$ has a unique index and its associated combination of process and resource. The transformational events are highlighted in bold. These events are effectively an *untimed* scheduled events list and are used to generate the healthcare delivery system Petri net firing vectors. Since the case does not explicitly detail dates and times, we estimate times and durations from average durations published in the literature to develop the scheduled event list visualized in Figure 6.

Now that the dynamics have been defined, an operating cost function can be calculated based on the firing vectors, $U^+[k]$, in the scheduled event list.

Definition 6 (Cumulative Operating Cost Function): Operating costs incur as transitions fire, representing the execution of capabilities in the healthcare delivery system. Given a capability cost vector, C of size $\sigma(\mathcal{E}_S)\times 1$, representing the cost for each capability and the input firing vector $U^+[k]$, the cumulative operating cost function, $\mathcal{C}$, is given by:

$$\mathcal{C}[k] = \sum_{k=1}^{t} C^T U^+[k], \tag{13}$$

A simple estimate of cost was made for each degree of freedom. Each transition is fired at a specific time and has an associated cost. The operating cost function, C was calculated by aggregating the cost for each degree of freedom capability as a function of time in days. The Orthopedic Case Cumulative Healthcare Delivery System Operating Cost is the top figure in Figure 4.

DOF Index	DOF Event	Process	Resource
35	e_{N2N2}	Transport from outside clinic to emergency dept.	by vehicle
2	e_{F1F2}	**Perform check in**	**by emergency dept.**
8	e_{M1F2}	Perform physical exam evaluation	by emergency dept.
6	e_{D1F2}	Decide on care planning	by emergency dept.
7	e_{D2F2}	Decide on care scheduling	by emergency dept.
29	e_{N10N1}	Transport from emergency dept. to radiology dept.	by transport staff & equip
27	e_{M2D}	Perform diagnostic testing	by radiology dept.
32	e_{N22N1}	Transport from radiology dept. to emergency dept.	by transport staff & equip
6	e_{D1F2}	Decide on care planning	by emergency dept.
7	e_{D2F2}	Decide on care scheduling	by emergency dept.
5	e_{F4F2}	**Perform therapeutic procedure**	**by emergency dept.**
3	e_{F2F2}	**Perform check out**	**by emergency dept.**
39	e_{N6N2}	Transport from emergency dept. to outside clinic	by vehicle
36	e_{N3F3}	Transport from outside clinic to orthopedic dept.	by vehicle
10	e_{F1F3}	**Perform check in**	**by orthopedic dept.**
16	e_{M1F3}	Perform physical exam evaluation	by orthopedic dept.
14	e_{D1F3}	Decide on care planning	by orthopedic dept.
15	e_{D2F3}	Decide on care scheduling	by orthopedic dept.
30	e_{N15N1}	Transport from orthopedic dept. to radiology dept.	by transport staff & equip
27	e_{M2D}	Perform diagnostic testing	by radiology dept.
33	e_{N23N1}	Transport from radiology dept. to orthopedic dept.	by transport staff & equip
14	e_{D1F3}	Decide on care planning	by orthopedic dept.
15	e_{D2F3}	Decide on care scheduling	by orthopedic dept.
12	e_{F3F3}	**Perform surgical procedure**	**by orthopedic dept.**
16	e_{M1F3}	Perform physical exam evaluation	by orthopedic dept.
14	e_{D1F3}	Decide on care planning	by orthopedic dept.
15	e_{D2F3}	Decide on care scheduling	by orthopedic dept.
11	e_{F2F3}	**Perform check out**	**by orthopedic dept.**
40	e_{N11N2}	Transport from orthopedic dept. to outside clinic	by vehicle
37	e_{N4N2}	Tranport from outside clinic to physical therapy	by vehicle
18	e_{F1F4}	**Perform check in**	**by physical therapy dept.**
23	e_{M1F4}	Perform evaluation physical exam	by physical therapy dept.
21	e_{D1F4}	Decide on care planning	by physical therapy dept.
22	e_{D2F4}	Decide on care scheduling	by physical therapy dept.
20	e_{F4F4}	**Peform therapeutic procedure**	**by physical therapy dept.**
19	e_{F2F4}	**Perform check out**	**by physical therapy dept.**
41	e_{N16N2}	Transport from physical therapy dept. to outside clinic	by vehicle

FIGURE 3. Acute Care Healthcare Delivery System Events based on the Orthopedic Case narrative described in terms of the Healthcare Delivery System Events found in J_S. The healthcare delivery transformational events are in bold.

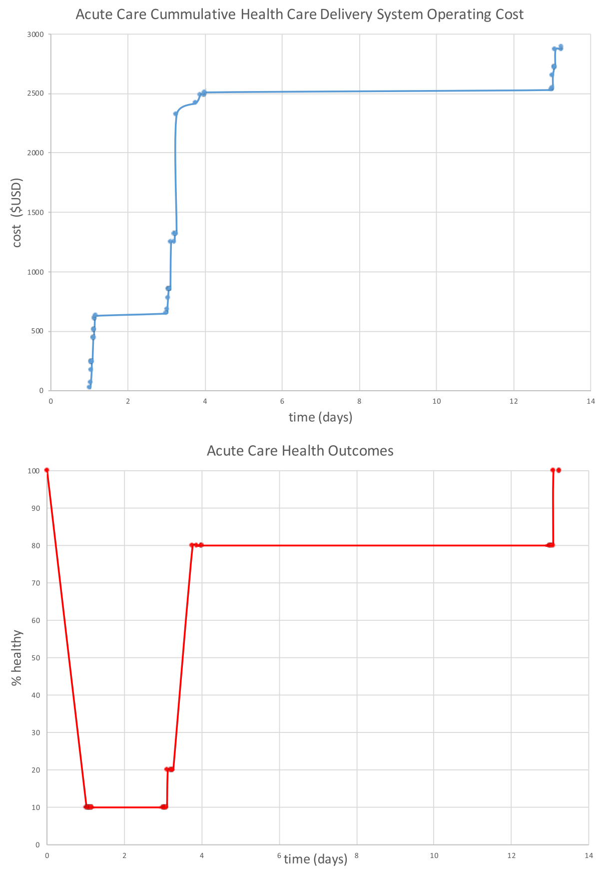

FIGURE 4. Acute Care Healthcare Delivery System Operating Cost in US dollars (top-blue) and Individual Health Outcome as % healthy (bottom-red) over a time period of 2 weeks. These figures are plotted together to highlight that while every transition firing has a cost, not every one of these fired transitions actually improves the individual's health outcome. While transporting, measuring, and deciding functions are critical to identifying and delivering treatment, typically when an individual is treated does their health outcome change.

B. THE CHRONIC CONDITION CARE ABSTRACTION

The healthcare delivery system model for the acute care example considered all of its inherent capabilities and integrated them within a Petri net model. For chronic care, however, several additional considerations are required. First, because chronic conditions continue well beyond a single visit to a healthcare facility, a resource entitled 'outside clinic' must be included in the model. Naturally, this will require the addition of transportation processes so as to enter and exit the clinic. Next, transportation degrees of freedom within the clinic are assumed to have a negligible duration and are therefore eliminated. K_S is modified accordingly. By Equation 1, the number of structural degrees of freedom changes as well. Consequently, a new projection operator $\mathbb{P}_C$ must be calculated such that:

$$\mathbb{P}_C(J_S \ominus K_S)^V = \mathbb{1}^{\sigma(\mathcal{E}_S)} \tag{14}$$

Finally, the resources within the clinic are aggregated by Equation 4 so as to yield to $\bar{\mathbb{R}}$ = {healthcare clinic, outside clinic}. Consequently, the healthcare delivery system Petri net incidence out and incidence in matrices become:

$$\mathcal{M}^- = A_R \sum_{y1=1}^{\sigma(\mathbb{R}_B)} e_{y1}^{\sigma(\mathbb{R}_B)} \left[\mathbb{P}_{\mathbb{C}}\left(X_{y_1}^-\right)^V\right]^T \tag{15}$$

$$\mathcal{M}^+ = A_R \sum_{y2=1}^{\sigma(\mathbb{R}_B)} e_{y2}^{\sigma(\mathbb{R}_B)} \left[\mathbb{P}_{\mathbb{C}}\left(X_{y_2}^+\right)^V\right]^T \tag{16}$$

This hierarchical aggregation implements the chronic condition care abstraction. The focus now becomes the various forms of transformation, decision, and measurement processes that the patient receives rather than transportation and queuing within the clinic.

C. HEALTH NET DYNAMICS

As mentioned previously, the Health Net is introduced so as to represent the clinical health state of individuals.

Definition 7 (Health Net [30]): Given an individual l_i, that is part of a population $L = \{l_1, \ldots, l_{\sigma(L)}\}$, the evolution of their clinical health state can be described as a fuzzy timed

Petri net [51]–[53]:

$$N_{l_i} = \{S_{l_i}, \mathcal{E}_{l_i}, \mathcal{M}_{l_i}, W_{l_i}, D_{l_i}, Q_{l_i}\} \tag{17}$$

where,

- N_{l_i} is the health net.
- S_{l_i} is the set of places describing a set of health states.
- $\mathcal{E}_{l_i}$ is the set of transitions describing health events.
- $\mathcal{M}_{l_i} \subseteq (S_{l_i} \times \mathcal{E}_{l_i}) \cup (\mathcal{E}_{l_i} \times S_{l_i})$ is the set of arcs describing the relations of (health states to health events) or (health events to health states).
- W_{l_i} is the set of weights on the arcs describing the health transition probabilities for the arcs.
- D_{l_i} is the set of transition durations.
- Q_{l_i} is the Petri net marking representing the likely presence of the set of health states as a discrete probabilistic state.

We now turn to model the health net for the individual, Adam, in orthopedic case study 1. Figure 5 visualizes the acute care individual health net. The health net shows Adam's original health state with a 'normal knee' that becomes damaged by his injury and results in pain and knee damage. He checks into the emergency department, which transforms him from an individual to a patient that can now receive healthcare services. Adam receives pharmacotherapy and equipment to manage the pain and reduce any further knee damage. He checks out and back into the clinic to receive surgery where his knee is repaired, but not completely functional. Finally, he becomes a patient again to receive physical therapy to heal his knee back to normal. After physical therapy, Adam's health state returns back to its original, normal health state. We explicitly include a 'patient' state to model *access* to healthcare systems. The health net shows

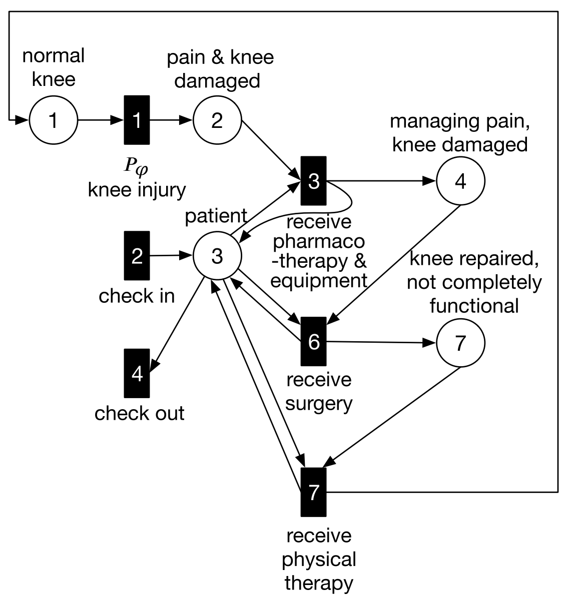

FIGURE 5. Acute Care Individual Health Net: visualizes the health state at the places *(circles)* and the health events, causing the changes in health state, at the transitions *(rectangles)*.

Adam's distributed *health states* at the places (circles) and his *health state transformations* or *health events* at the transitions (rectangles). These occur due to healthcare delivery system events P_F or stochastic human processes P_φ. As is common with acute conditions, there is a serial progression of events which when successful return the patient back to a normal health state.

The Petri net structure leads directly to the definition of its discrete-event dynamics.

Definition 8 (Fuzzy Timed Petri Net (Discrete-Event) Dynamics [30], [54]): Given a binary input firing vector $U^+_{l_i}[k]$ and a binary output firing vector $U^-_{l_i}[k]$ both of size $\sigma(\mathcal{E}_{l_i}) \times 1$, and the positive and negative components $\mathcal{M}^+_{l_i}$ and $\mathcal{M}^-_{l_i}$ of the Petri net incidence matrix of size $\sigma(S_{l_i}) \times \sigma(\mathcal{E}_{l_i})$, the evolution of the marking vector Q_{l_i} is given by the state transition function $\Phi(Q_{l_i}[k], U_{l_i}[k])$:

$$Q_{l_i}[k+1] = \Phi(Q_{l_i}[k], U^-_{l_i}[k], U^+_{l_i}[k]) \tag{18}$$

where $Q_{l_i} = [Q_{Sl_i}; Q_{\mathcal{E}l_i}]$ and

$$Q_{Sl_i}[k+1] = Q_{Sl_i}[k] + M^+_{l_i} U^+_{l_i}[k] - M^-_{l_i} U^-_{l_i}[k] \tag{19}$$

$$Q_{El_i}[k+1] = Q_{El_i}[k] - U^+_{l_i}[k] + U^-_{l_i}[k] \tag{20}$$

Q_{Sl_i} is introduced to probabilistically mark Petri net places whereas Q_{El_i} is introduced to mark the likelihood that a timed transition is currently firing. The transitions are fired based on a scheduled event list that combines the discrete events with a time interval as described in Definition 5.

The synchronized dynamics of the healthcare delivery system Petri net and the individual net are shown in Figure 6 as two scheduled event lists side-by-side. Thus, the entirety of the simulation model can be calculated using the two Petri nets, and specifically the state transition functions.

Now that the health net and its dynamics are defined. An individual's health outcome function can be calculated.

Definition 9 (Health Outcome Function): An individual's health outcome is represented by the set of places describing health state, S_{l_i}. Given a value vector that numerically represents health state, V of size $\sigma(S_{l_i}) \times 1$, and the health state vector $Q_{Sl_i}[k]$, the health outcome function, $\mathcal{H}_{l_i}$, is given by:

$$\mathcal{H}_{l_i}[k] = V^T Q_{Sl_i}[k] \tag{21}$$

For the orthopedic case, each health state in the health net was assigned a numerical value based on an estimate level of knee functionality. The bottom of Figure 4 visualizes the Health Outcome Function relative to time in days. These figures are plotted together to highlight that while every transition firing has a cost, not every one of these fired transitions actually improves the individual's health outcome. While transporting, measuring, and deciding functions are critical to identifying and delivering treatment, typically when an individual is treated does their health outcome change. In addition, for the acute care setting, an individual's health state can be returned to near complete health.

Individual/Patient							Healthcare Delivery System				
Transitions				Places			Places	Transitions			
Transition Description	Health Value (%)	Duration (days)	Q_{EI}	Q_{SI}	k	Time (days)	Q_S	Q_{ES}	Duration (days)	Cost ($)	Transition Description
	100			1	1	0	1			0	
stochastic human process: knee injury		0.0003	1	2	2	1				0	
	10	0.02		2	3	1.02	2	35	0.02	20	Transport from outside clinic to emergency dept. by vehicle
transform individual to patient	**10**	**0.02**	**2**	**2,3**	**4**	**1.04**	**2**	**2**	**0.02**	**50**	**Perform check in by emergency dept.**
	10	0.02		2,3	5	1.06	2	8	0.02	100	Perform physical exam evaluation by emergency dept.
	10	0.01		2,3	6	1.061	2	6	0.01	70	Decide on care planning by emergency dept.
	10	0.01		2,3	7	1.071	2	7	0.01	0	Decide on care scheduling by emergency dept.
	10	0.02		2,3	8	1.081	5	29	0.02	0	Transport from emergency dept. to radiology dept. by transport staff & equip
	10	0.02		2,3	9	**1.101**	5	27	0.02	200	Perform diagnostic testing by radiology dept.
	10	0.02		2,3	10	1.121	2	32	0.02	0	Transport from radiology dept. to emergency dept. by transport staff & equip
	10	0.01		2,3	11	1.122	2	6	0.01	70	Decide on care planning by emergency dept.
	10	0.01		2,3	12	1.132	2	7	0.01	0	Decide on care scheduling by emergency dept.
receive pharmacotherapy & equip.	**10**	**0.01**	**3**	**4,3**	**13**	**1.142**	**2**	**5**	**0.01**	**100**	**Perform therapeutic procedure by emergency dept.**
transform patient to individual	**10**	**0.02**	**4**	**4**	**14**	**1.152**	**2**	**3**	**0.02**	**0**	**Perform check out by emergency dept.**
	10	0.01		4	15	1.172	1	39	0.01	20	Transport from emergency dept. to outside clinic by vehicle
	10	0.02		4	16	3	3	36	0.02	20	Transport from outside clinic to orthopedic dept. by vehicle
transform individual to patient	**10**	**0.02**	**2**	**4,3**	**17**	**3.02**	**3**	**10**	**0.02**	**30**	**Perform check in by orthopedic dept.**
	10	0.02		4,3	18	3.04	3	16	0.02	100	Perform physical exam evaluation by orthopedic dept.
	10	0.04		4,3	19	**3.06**	3	14	0.04	70	Decide on care planning by orthopedic dept.
	10	0.02		4,3	20	3.1	3	15	0.02	0	Decide on care scheduling by orthopedic dept.
	10	0.02		4,3	21	3.101	5	30	0.02	0	Transport from orthopedic dept. to radiology dept. by transport staff & equip
	20	0.08		4,3	22	3.121	5	27	0.08	400	Perform diagnostic testing by radiology dept.
	20	0.02		4,3	23	3.201	3	33	0.02	0	Transport from radiology dept. to orthopedic dept. by transport staff & equip
	20	0.04		4,3	24	3.221	3	14	0.04	70	Decide on care planning by orthopedic dept.
	20	0.02		4,3	25	3.261	3	15	0.02	0	Decide on care scheduling by orthopedic dept.
receive surgery	**20**	**0.5**	**6**	**7,3**	**26**	**3.262**	**3**	**12**	**0.5**	**1000**	**Perform surgical procedure by orthopedic dept.**
	80	0.12		7,3	27	3.762	3	16	0.12	100	Perform physical exam evaluation by orthopedic dept.
	80	0.1		7,3	28	3.882	3	14	0.1	70	Decide on care planning by orthopedic dept.
	80	0.02		7,3	29	3.982	3	15	0.02	0	Decide on care scheduling by orthopedic dept.
transform patient to individual	**80**	**0.02**	**4**	**7**	**30**	**4.002**	**3**	**11**	**0.02**	**0**	**Perform check out by orthopedic dept.**
	80	0.01		7	31	4.003	1	40	0.01	20	Transport from orthopedic dept. to outside clinic by vehicle
	80	0.02		7	32	13	4	37	0.02	20	Transport from outside clinic to physical therapy by vehicle
transform individual to patient	**80**	**0.02**	**2**	**7,3**	**33**	**13.02**	**4**	**18**	**0.02**	**20**	**Perform check in by physical therapy dept.**
	80	0.04		7,3	34	13.021	4	23	0.04	100	Perform evaluation physical exam by physical therapy dept.
	80	0.02		7,3	35	13.061	4	21	0.02	70	Decide on care planning by physical therapy dept.
	80	0.01		7,3	36	13.081	4	22	0.01	0	Decide on care scheduling by physical therapy dept.
receive physical therapy	**100**	**0.15**	**7**	**1,3**	**37**	**13.091**	**4**	**20**	**0.15**	**150**	**Perform therapeutic procedure by physical therapy dept.**
transform patient to individual	**100**	**0.02**	**4**	**1**	**38**	**13.241**	**4**	**19**	**0.02**	**0**	**Perform check out by physical therapy dept.**
	100	0.01		1	39	13.242	1	41	0.01	20	Transport from physical therapy dept. to outside clinic by vehicle

FIGURE 6. Acute Care Dynamics of the Two Petri nets: the healthcare delivery system Petri net and the individual net are now synchronized. The scheduled events of each Petri net are shown side by side.

D. COORDINATION OF THE HEALTHCARE DELIVERY SYSTEM PETRI NET & INDIVIDUAL HEALTH NET DYNAMICS

As expected, the healthcare delivery system Petri net and the health net dynamics are inherently coupled. Each transformation process in the healthcare delivery system induces its corresponding health event. For each individual, l_i, this feasibility condition can be captured in a binary individual transformation feasibility matrix [30].

Definition 10 (Individual Transformation Feasibility Matrix Λ_{F_i} [30], [39]–[46]): a binary matrix of size $\sigma(\mathcal{E}_{l_i}) \times \sigma(P_F)$, where $\Lambda_{F_i}(x, j) = 1$ if transformational process p_{F_j} realizes the health event e_{xl_i}.

An individual firing matrix is introduced to synchronize the healthcare delivery system Petri net firing vectors with those of the (individual) health nets.

Definition 11 (Individual Health Firing Matrix [45], [55]): A binary individual health firing matrix $\mathcal{U}[k]$ of size $\sigma(\mathcal{E}_S) \times \sigma(L)$, whose element $u_{l_i\psi,l}[k] = 1$ when the k^{th} firing timing triggers an individual l to take structural degree of freedom ψ for action.

Consequently, the healthcare delivery system input firing vectors at a given moment k become [45]

$$U^- = \mathcal{U}\mathbb{1}^{\sigma(L)} \tag{22}$$

and each health net firing vector at a given moment k becomes [45]

$$\Lambda_{F_i}^T \cdot U_{l_i} = \mathcal{A}_F \cdot \mathcal{U} \cdot e_{l_i}^{\sigma(L)T} \tag{23}$$

and $\mathcal{A}_F$ serves to select out the structural degrees of freedom associated with transformation. For an extensive discussion of the synchronization, the reader is referred to [29] and [56].

To complete the model, the healthcare delivery system Petri net and individual health net must be coordinated. The individual transformation feasibility matrix, shown in Figure 7, is constructed by linking the individual health net transitions (i.e., health events) to the corresponding healthcare delivery system transformational events (i.e., transformation process P_F).

The mathematical model was implemented in MATLAB comprised of Equations 11-12 and 19-20. As the incidence matrices are tedious to construct manually, we developed

Individual Health State Transformations (Health Events) \ Healthcare Delivery System Transformational Events (P_F)	e_{F1} Perform check in (DOF=1)	e_{F2} Perform check out (DOF=2)	e_{F4} Perform therapeutic procedure (DOF=3)	e_{F3} Perform surgical procedure (DOF=4)
transform from individual to patient (Transition=2)	■			
transform from patient to individual (Transition=4)		■		
receive pharmacotherapy & equipment (Transition=3)			■	
receive surgery (Transition=6)				■
receive physical therapy (Transition=7)			■	

FIGURE 7. **Acute Care Individual Transformation Feasibility Matrix Λ_F.**

a toolbox to construct these matrices automatically [57]. The HFGT toolbox is freely and openly available on GitHub at https://github.com/LIINES/HFGTToolbox. We have demonstrated that the HFGT toolbox has the potential to handle significant computation complexity with millions of places and transitions [58].

IV. CHRONIC CARE ILLUSTRATIVE EXAMPLE

In contrast to the previous example, a neuro-oncology case study is chosen to demonstrate the model's applicability to chronic conditions. Section IV-A presents the full narrative of the case as presented by Park et. al. [59]. Section IV-B then presents the healthcare delivery system model; first as a knowledge base, then in terms of a list of events, and finally as a Petri net. Next, Section IV-C presents the health net. Finally, Section IV-D presents the coordination of the healthcare delivery system Petri net and individual health net dynamics.

A. DESCRIPTION OF NEURO-ONCOLOGY CASE

Case Study 2: 'The patient was a 32-year-old, right-handed woman without significant past medical history who presented for evaluation of headaches and intermittent short-term memory loss. She also reported mild nausea, but was otherwise asymptomatic. Her headaches had begun approximately 3 *months before her presentation. On neurologic examination, no deficit was appreciated. She underwent a head CT with the finding of a large, poorly enhancing right occipital-parietal mass that appeared to be located within the lateral ventricle. Subsequent MR imaging scanning confirmed the location of the tumor in the trigone with local expansion of the ventricle. Similar to the CT, minimal enhancement was noted. On the basis of the imaging characteristics, a low-grade astrocytoma was felt to be the most likely diagnosis. In light of the size of the tumor and its location, a parietooccipital surgical approach was performed. The tumor appeared grayish and was predominantly firm, necessitating piecemeal removal. The tumor was not particularly vascular, and a distinct plane between tumor and ependyma was identified, but there were several areas where the tumor appeared to infiltrate into adjacent brain parenchyma. Frozen-section pathologic analysis was described as abnormal and cellular but was not specifically diagnostic. Tumor resection was continued until a near-total removal was accomplished. Postoperatively, the patient remained without neurologic deficit. Follow-up MR imaging showed near-total removal of the tumor. Histologic sections were examined by light microscopy. The neoplasm was hypercellular with necrosis and endothelial cell proliferation, hallmarks of GBM, World Health Organization (WHO) classification grade IV (9, 10). Gemistocytes were distributed throughout the neoplasm with rare mitoses. These cells had hyaline, eosinophilic cytoplasm, and eccentric, hyperchromatic nuclei, some of which were large and pleomorphic. Immunostains for glial fibrillary acidic protein (GFAP), S-100 protein, vimentin, and neurofilament were positive, whereas immunostains for muscle-specific actin, alpha smooth muscle, and synaptophysin were negative. MIB-1 was positive, with a low proliferation index. The morphology and GFAP positivity suggested a diagnosis of diffuse gemistocytic astrocytoma; however, the tumor necrosis and microvascular proliferation raised the tumor grade to grade IV GBM. Because of the unusual radiographic appearance and location, the histologic slides were also reviewed by physicians at the Armed Forces Institute of Pathology (Bethesda, MD), who confirmed the initial diagnosis. Whole-brain radiation and chemotherapy were subsequently instituted. At 2-year follow-up, the patient complained of headaches but remained neurologically intact with no evidence of tumor progression on MR imaging.' [59]*

B. MODELING THE HEALTHCARE DELIVERY SYSTEM OF THE NEURO-ONCOLOGY CASE

The chronic case study text is interpreted so as to identify the healthcare delivery system processes and resources. They are modeled at a level of aggregation typical of clinical narratives. For example, the narrative states '...a parietoccipital surgical approach was performed' [59]. Here, the healthcare delivery system process is aggregated to 'Perform surgery' and the 'neuro surgery & oncology departments' resources describes the aggregation of human and technical resources. Again, an 'outside clinic' resource is included to reflect the individual entering and exiting the healthcare delivery system in the case's six clinical visits. As discussed in Section III-B, the transportation capabilities within the clinic are assumed to be always available, of relatively short duration, and of sufficient capacity. They are eliminated from the knowledge base so as to focus on the more valuable healthcare delivery capabilities of transformation, decision and measurement. Therefore, the transportation processes are reduced to

'Transport from outside to inside clinic' and 'Transport from inside to outside clinic'. Furthermore, given the long term nature of this example, the decision processes of care planning and care scheduling are combined into 'Decide on neurological/oncological care'.

The resources and processes are classified as either transformation, decision, measurement or transportation and used to construct the system knowledge base J_S shown in Figure 8. For simplicity, the system is assumed to not have any event constraints, $K_S = 0$. The system availability matrix and consequently the structural degrees of freedom can be calculated using Equation 1 such that $DOF_S = 9$.

		Resources: R_{F1} outside clinic	R_{F2} neuro surgery & oncology departments	R_N vehicle
P_{F1}	Perform check in		1	
P_{F2}	Perform check out		2	
P_{F3}	Perform surgical resection		3	
P_{F4}	Perform radiation & chemotherapy treatment		4	
P_D	Decide on neurological/ oncological care		5	
P_{M1}	Assess neurological/ oncological symptoms		6	
P_{M2}	Perform diagnostic testing		7	
P_{N1}	Transport from F1 to F2			8
P_{N2}	Transport from F2 to F1			9

FIGURE 8. Chronic Care Healthcare Delivery System Knowledge Base J_S with allocated processes to resources (dark filled). Filled elements represent allocated processes to resources. Since we identified no constraints (i.e., $K_S = 0$, J_S is also equal to A_S. Thus, the enumerated number in the black box indicates the index for each degree of freedom.

DOF Index	DOF Event	Process	Resource
8	e_{N1N}	Transport from outside to inside clinic	by vehicle
1	e_{F1F2}	**Perform check in**	**by neuro surg/onc depts.**
6	e_{M1F2}	Assess neurological/oncological symptoms	by neuro surg/onc depts.
5	e_{DF2}	Decide on neurological/oncological care	by neuro surg/onc depts.
7	e_{M2F3}	Perform diagnostic testing	by neuro surg/onc depts.
2	e_{F2F2}	**Perform check out**	**by neuro surg/onc depts.**
9	e_{N2N}	Transport from inside to outside clinic	by vehicle
8	e_{N1N}	Transport from outside to inside clinic	by vehicle
1	e_{F1F2}	**Perform check in**	**by neuro surg/onc depts.**
7	e_{M2DM}	Perform diagnostic testing	by neuro surg/onc depts.
2	e_{F2F2}	**Perform check out**	**by neuro surg/onc depts.**
9	e_{N2N}	Transport from inside to outside clinic	by vehicle
8	e_{N1N}	Transport from outside to inside clinic	by vehicle
1	e_{F1F2}	**Perform check in**	**by neuro surg/onc depts.**
5	e_{DF2}	Decide on neurological/oncological care	by neuro surg/onc depts.
2	e_{F2F2}	**Perform check out**	**by neuro surg/onc depts.**
9	e_{N2N}	Transport from inside to outside clinic	by vehicle
8	e_{N1N}	Transport from outside to inside clinic	by vehicle
1	e_{F1F2}	**Perform check in**	**by neuro surg/onc depts.**
3	e_{F3F2}	**Peform surgical resection**	**by neuro surg/onc depts.**
6	e_{M1F2}	Assess neurological/oncological symptoms	by neuro surg/onc depts.
5	e_{DF2}	Decide on neurological/oncological care	by neuro surg/onc depts.
7	e_{M2F2}	Perform diagnostic testing	by neuro surg/onc depts.
5	e_{DF2}	Decide on neurological/oncological care	by neuro surg/onc depts.
2	e_{F2F2}	**Perform check out**	**by neuro surg/onc depts.**
9	e_{N2N}	Transport from inside to outside clinic	by vehicle
8	e_{N1N}	Transport from outside to inside clinic	by vehicle
1	e_{F1F2}	**Perform check in**	**by neuro surg/onc depts.**
4	e_{F4F2}	**Perform radiation & chemotherapy treatment**	**by neuro surg/onc depts.**
6	e_{M1F2}	Assess neurological/oncological symptoms	by neuro surg/onc depts.
5	e_{DF2}	Decide on neurological/oncological care	by neuro surg/onc depts.
2	e_{F2F2}	**Perform check out**	**by neuro surg/onc depts.**
9	e_{N2N}	Transport from inside to outside clinic	by vehicle
8	e_{N1N}	Transport from outside to inside clinic	by vehicle
1	e_{F1F2}	**Perform check in**	**by neuro surg/onc depts.**
7	e_{M2F2}	Perform diagnostic testing	by neuro surg/onc depts.
5	e_{DF2}	Decide on neurological/oncological care	by neuro surg/onc depts.
2	e_{F2F2}	**Perform check out**	**by neuro surg/onc depts.**
9	e_{N2N}	Transport from inside to outside clinic	by vehicle

FIGURE 9. Chronic Care Healthcare Delivery System Events based on the Neuro-Oncology Case narrative in terms of the Healthcare Delivery System Events found in J_S. The healthcare delivery transformational process events are in bold.

The Case Study 2 narrative is then rewritten as a string of healthcare delivery system events $\mathcal{E}_S$ as shown in Figure 9. Each event in $\mathcal{E}_S$ has a unique index and its associated combination of process and resource. The transformational events are highlighted in bold. These events are effectively an *untimed* scheduled events list and are used to generate the healthcare delivery system Petri net firing vectors.

Finally, the healthcare delivery system Petri net is constructed. As discussed in Section III-B, the chronic care abstraction is utilized to abstract the many healthcare delivery system resources to a single 'healthcare clinic' resource. Figure 10 shows the Petri net of the healthcare delivery system superimposed on a light-grey physical layout representing the whole clinic.

C. MODELING THE INDIVIDUAL HEALTH NET OF THE NEURO-ONCOLOGY CASE

The chronic case study narrative and its associated healthcare delivery system events now serve to determine the health net for the neuro-oncology patient as shown in Figure 11. The health states are identified based on typical clinical outcomes of the healthcare transformational processes: 'Perform surgical resection' and 'Perform radiation & chemotherapy treatment'. The transitions are defined completely based on the human body. Transitions can be the human body response to healthcare delivery system transformational events, or the human body's predisposition to a condition or illness, the human body's progression through a disease, or the human body's ability to heal.

Because the health net is fuzzy, the outcome of any given transition is probabilistic. For example, after a tumor resection three different outcomes are possible to describe the extent of resection: gross-total resection [GTR], near-total resection [NTR] and sub-total resection [STR]). For simplicity, the probabilities of these outcomes are assumed to be equal. In practice, the probabilities would be captured from the medical literature or validated with clinical data of surgery outcomes. Next, three transitions occur in parallel to

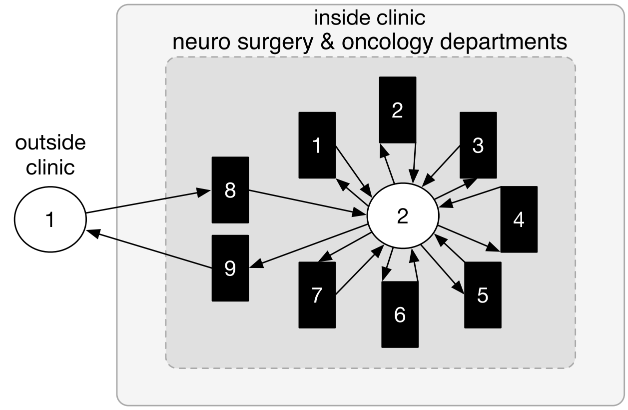

FIGURE 10. Chronic Care Healthcare Delivery System Petri net. The places (*circles*), represent 1='outside clinic' and 2='healthcare clinic'. The transitions (*rectangles*), are numbered based on the healthcare delivery system structural degrees of freedom index ψ_i.

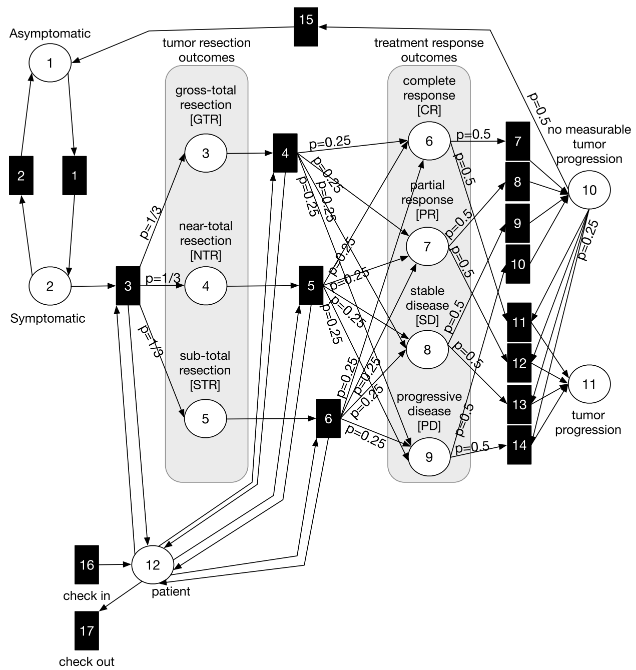

FIGURE 11. Chronic Care Individual Net: visualizes the health states at the places *(circles)* and the health events at the transitions *(rectangles)* representing the stochastic human process or the healthcare delivery system transformational process firing.

represent radiation & chemotherapy of the tumor/cancer in its current condition. This leads to four possible treatment response outcomes based on the McDonald criterion [60] (i.e., complete response [CR], partial response [PR], stable disease [SD], progressive disease [PD]). These lead to states associated with further tumor progression, or no measurable tumor progression followed by an asymptomatic state. In this case, the health net, as a fuzzy-timed Petri net, not only shows the dynamic evolution of an individual's distributed health state in the clinical sense but its stochastic nature lends itself to Bayesian statistics.

In addition, the health net can be used to model distributed health states. For example, the discussion above describes the capture of health state as defined by clinical medicine (e.g., near-total resection with stable disease). The model was constructed to capture access to care by specifically modeling the individual as a patient when they are in the clinic. Only then can the individual receive healthcare services. This highlights the choice of using a Petri net to mathematically model the dynamic and distributed health states for the individual.

D. MODELING THE COORDINATION OF THE HEALTHCARE DELIVERY SYSTEM PETRI NET & INDIVIDUAL HEALTH NET OF THE NEURO-ONCOLOGY CASE

To complete the model, the healthcare delivery system Petri net and the individual health net must be coordinated. The individual transformation feasibility matrix, shown in Figure 12, is constructed by linking the individual health net transitions (i.e., health events) to the corresponding healthcare delivery system transformational events (i.e., transformation process P_F). Note that the radiation and chemotherapy transformation process is tied to three health events; not just one. This is because the individual's health evolves differently to the same stimulus depending on their current condition.

Healthcare Delivery System Transformational Events (P_F)

Individual Health State Transformations (Health Events)	e_{F1} Perform check in (DOF=1)	e_{F2} Perform check out (DOF=2)	e_{F3} Perform surgical resection (DOF=3)	e_{F4} Perform radiation & chemotherapy treatment (DOF=4)
transform from individual to patient (Transition=15)	■			
transform from patient to individual (Transition=16)		■		
brain cancer illness (Transition=1)				
healing from symptoms (Transition=2)				
receive surgical resection (Transition=3)			■	
receive radiation & chemotherapy with GTR (Transition=4)				■
receive radiation & chemotherapy with NTR (Transition=5)				■
receive radiation & chemotherapy with STR (Transition=6)				■
no progression of disease (Transition=7)				
progression of disease (Transition=8)				
healing from treated brain tumor (Transition=9)				

FIGURE 12. Chronic Care Individual Transformation Feasibility Matrix Λ_F.

The synchronized dynamics of the healthcare delivery system Petri net and the individual net are shown in Figure 13 as two scheduled event lists side by side.

Finally, the healthcare delivery system operating cost and individual health outcome dynamics for this Neuro-Oncology Chronic case can be shown over time (days) in Figure 14. Again, plotting the figures together highlights that while every transition firing has a cost, transporting, measuring, and

Individual/Patient							Healthcare Delivery System				
Transitions				Places			Places	Transitions			
Transition Description	Health Value %	Duration (days)	Q_{EI}	Q_{SI}	k	Time (days)	Q_S	Q_{ES}	Duration (days)	Cost ($)	Transition Description
	100			1	0	0	1			0	
stochastic human process: headaches++		90	1	2	1	1	1			0	
	30	0.02		2	2	91	1	8	0.02	20	Transport from outside to inside clinic by vehicle
transform individual to patient	30	0.02	16	2,12	4	91.02	2	1	0.02	30	Perform check in by neuro surg/onc depts.
	30	0.04		2,12	6	91.04	2	6	0.04	70	Assess neurological/oncological symptoms by neuro surg/onc depts.
	30	0.04		2,12	8	91.08	2	5	0.04	100	Decide on neurological/oncological care by neuro surg/onc depts.
	30	0.08		2,12	10	91.12	2	7	0.08	200	Perform diagnostic testing by neuro surg/onc depts.
transform patient to individual	30	0.02	17	2,12	12	95	2	2	0.02	0	Perform check out by neuro surg/onc depts.
	30	0.01		2	14	95.021	1	9	0.01	20	Transport from inside to outside clinic by vehicle
	30	0.02		2	16	95.031	1	8	0.02	20	Transport from outside to inside clinic by vehicle
transform individual to patient	30	0.02	16	2,12	18	95.051	2	1	0.02	30	Perform check in by neuro surg/onc depts.
	30	0.02		2,12	20	95.071	2	7	0.02	200	Perform diagnostic testing by neuro surg/onc depts.
transform patient to individual	30	0.02	17	2,12	22	95.091	2	2	0.02	0	Perform check out by neuro surg/onc depts.
	30	0.01		2	24	95.111	1	9	0.01	20	Transport from inside to outside clinic by vehicle
	30	0.02		2	26	100	1	8	0.02	20	Transport from outside to inside clinic by vehicle
transform individual to patient	30	0.02	16	2,12	28	100.02	2	1	0.02	30	Perform check in by neuro surg/onc depts.
	60	0.02		2,12	29	100.04	2	5	0.02	70	Decide on neurological/oncological care by neuro surg/onc depts.
transform patient to individual	60	0.01	17	2,12	30	100.06	2	2	0.01	0	Perform check out by neuro surg/onc depts.
	60	0.01		2	31	100.07	1	9	0.01	20	Transport from inside to outside clinic by vehicle
	60	0.02		2	32	100.08	1	8	0.02	20	Transport from outside to inside clinic by vehicle
transform individual to patient	60	0.02	16	2,12	33	100.1	2	1	0.02	30	Perform check in by neuro surg/onc depts.
receive surgical (near-total) resection	60	0.02	3	4,12	34	100.12	2	3	0.95	2000	Peform surgical resection by neuro surg/onc depts.
	60	0.1		4,12	35	101.07	2	6	0.1	200	Assess neurological/oncological symptoms by neuro surg/onc depts.
	60	0.04		4,12	36	101.17	2	5	0.04	70	Decide on neurological/oncological care by neuro surg/onc depts.
	60	0.04		4,12	37	101.21	2	7	0.04	200	Perform diagnostic testing by neuro surg/onc depts.
	60	0.04		4,12	38	101.25	2	5	0.04	70	Decide on neurological/oncological care by neuro surg/onc depts.
transform patient to individual	60	0.02	17	4,12	39	101.29	2	2	0.02	0	Perform check out by neuro surg/onc depts.
	60	0.01		4	40	101.31	1	9	0.01	20	Transport from inside to outside clinic by vehicle
	60	0.02		4	41	110	1	8	0.02	20	Transport from outside to inside clinic by vehicle
transform individual to patient	60	0.02	16	4,12	42	110.02	2	1	0.02	30	Perform check in by neuro surg/onc depts.
receive radiation & chemo w/ a stable	80	0.5	5	8,12	43	110.04	2	4	0.5	3000	Perform radiation & chemotherapy treatment by neuro surg/onc depts.
	80	0.04		8,12	44	110.54	2	6	0.04	200	Assess neurological/oncological symptoms by neuro surg/onc depts.
	80	0.02		8,12	46	110.58	2	5	0.02	70	Decide on neurological/oncological care by neuro surg/onc depts.
transform patient to individual	80	0.02	17	8,12	48	110.6	2	2	0.02	0	Perform check out by neuro surg/onc depts.
	80	0.01		8	49	110.62	1	9	0.01	20	Transport from inside to outside clinic by vehicle
stochastic human process: headaches++	50	20	9			810.62	1				
	50	0.02		10	50	830.62	1	8	0.02	20	Transport from outside to inside clinic by vehicle
transform individual to patient	50	0.02	16	10,12	51	830.64	2	1	0.02	30	Perform check in by neuro surg/onc depts.
	50	0.1		10,12	52	830.66	2	7	0.1	200	Perform diagnostic testing by neuro surg/onc depts.
	50	0.04		10,12	53	830.76	2	5	0.04	70	Decide on neurological/oncological care by neuro surg/onc depts.
transform patient to individual	50	0.01	17	10,12	54	830.8	2	2	0.01	0	Perform check out by neuro surg/onc depts.
	50	0.01		10	55	830.81	1	9	0.01	20	Transport from inside to outside clinic by vehicle

FIGURE 13. Chronic Care Dynamics of the Two Petri nets: the healthcare delivery system Petri net and the individual net are now synchronized. The scheduled events of each Petri net are shown side by side.

deciding processes may not improve health outcome. Furthermore, for the chronic care setting, while an individual's health state may be improved, they do not typically return to complete health. In addition, health outcomes for chronic conditions tend to decline over time.

V. RESULTS & DISCUSSION

The illustrative examples demonstrate the dynamic model for personalized healthcare delivery and managed health outcomes. Such a model has several important aspects. It applies equally to the care of both acute and chronic conditions, it transparently describes health outcomes, and it transparently links cost and outcomes.

First, the two illustrative examples demonstrate the model's applicability to both acute and chronic care. The acute care dynamic modeling resembles those commonly found in industrial engineering and operations research literature [61]–[64]. It emphasizes the importance of scheduling and minimized queuing. At the timescale of a clinic visit, acute care decision processes like care planning and scheduling are critical. The timeliness of decision-making was highlighted in the acute care of the orthopedic case. Acute care requires a more granular resolution of healthcare delivery system capabilities (i.e., structural degrees of freedom). Consequently, many more are utilized per visit relative to chronic care. The utilization of spatially distributed transformative, decision, and measurement capabilities within a short period of time naturally raises questions of transportation (e.g., in emergency rooms) and queues (e.g., in patient care). In chronic care, these concerns are diminished. The model abstracts away transportation within the clinic so as to focus on the coordination of transformation, decision and measurement processes.

The health net in this model is an important contribution that serves to transparently describe health outcomes. In acute care, the health net tends to cycle back to an initially healthy state in a fairly short period of time; and perhaps within a single visit. In chronic care, not only are multiple clinical

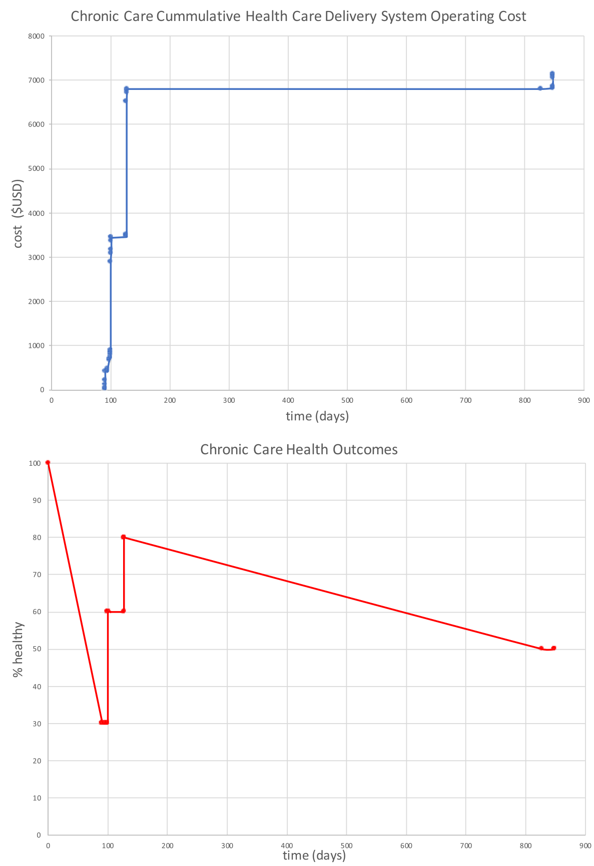

FIGURE 14. **Chronic Care Healthcare Delivery System Operating Cost in US dollars (top-blue) and Individual Health Outcome as % healthy (bottom-red) over a time period of more than 2 years (≈900 days).**

visits required but the state of the individual's health must be tracked in the meantime. While in some chronic conditions a return to a healthy state is possible, in most instances the healthcare delivery system must actively track and manage its degradation.

Although both the acute and chronic care examples included several visits, we describe a mathematical criterion for differentiating between chronic and acute care. The criterion is a ratio, where the numerator is the typical duration of a visit and the denominator is the typical duration of an event associated with the spontaneous evolution of the illness. In the acute care example, the ratio is more than one and can be relatively large. In the orthopedic acute example, a visit was on the order of a day and the spontaneous event leading to the knee injury was on the order of several seconds. This leads to a very large ratio (1day/0.0003day=3,333). Conversely, in the neuro-oncology chronic example, a healthcare visit was on the order of 1-2 days, while the spontaneous evolution of the illness was on the order of months to years. Even a conservative calculation leads to a very small number less than one (2days/90days=0.022).

Indeed, the most important aspect of the model is its coherence between the healthcare delivery system and the individual's health state. The states of the Petri nets are tied directly and should ideally be coordinated in order to deliver effective care. Whether for acute or chronic conditions, time is of the essence. Because the health nets have stochastic processes that will fire spontaneously, the healthcare delivery system must take timely and coordinated action to avoid adverse and negative health outcomes.

Finally, it is important to recognize that each healthcare delivery system degree of freedom incurs a cost every time it is fired. Therefore, as the two Petri nets evolve simultaneously, the discrete event simulation transparently reveals the accumulation of incurred cost versus the evolution of health outcomes as shown in Figures 4 and 14.

VI. CONCLUSION & FUTURE WORK

In conclusion, this paper develops a dynamic system model for personalized healthcare delivery and managed individual health outcomes. The dynamics of the model rests upon the developed systems architecture from prior work. This work draws upon a hetero-functional graph theory, an ontological convergence of model-based systems engineering and network science, along with Petri nets. The dynamic model coordinates the healthcare delivery system and individual net. The healthcare delivery net evolves as the transitions fire when the system is utilized, while the individual net evolves as the individual's health state evolves due to the spontaneous firing of stochastic process and as the individual receives transformative processes by the healthcare delivery system. The dynamic model was then demonstrated for acute and chronic care examples to show the versatility of the model. Furthermore, the dynamic model was used to produce two parametric functions of time dynamically showing healthcare delivery system operating cost over time and patient outcome values over time. These could be used to understand what healthcare capability utilizations or utilization patterns lead to better patient outcome values.

The development of the model opens several new avenues for future work. The Petri net firing vectors indicated as inputs to the model provide an opportunity for the development of rigorous decision-making algorithms. Second, the model may be applied to new case studies of potentially larger scope. The clear trade-offs between cost and health outcomes is likely to be of interest to many healthcare delivery system stakeholders including clinicians, healthcare facility administrators, insurance companies, and regulators.

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